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DECONFINEMENT OF QUARKS IN THE NAMBU-GOTO STRING WITH MASSIVE ENDS

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It is shown that the critical (or deconfinement) temperature for the Nambu-Goto string connecting the point-like masses (quarks) does not depend on the value of these masses and it is the same as that in the case of the string with fixed ends (infinitely heavy immobile quarks).

1 Introduction

We were lucky having collaborated with Professor Hagen Kleinert during our studies in string models. Prominent scientific achievements reached by him are well known but their presentation calls for a separate article. Here we would like to note briefly our impression from the personal contacts with him. The high incontestable authority of Professor Kleinert is harmoniously supplemented with his personal fascination. He literally "charges" colleagues by his creative energy. He is always full of ideas and new approaches to the problems under consideration and he readily shares those with his collaborators. To any question addressed to Professor H. Kleinert, one receives a clear-cut answer that testifies a profound understanding of the problem at hand. Kleinert's merry and slightly ironical temper renders the personal contacts with him easy and nice without any hint of pressure of his authority.

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We have decided to present in this book the results of a direct continuation of our joint work with Professor H. Kleinert devoted to string dynamics [1]. In Refs. [2-4], the dependence of the interquark potential on the quark masses has been investigated in the model of the relativistic string with ends loaded by point-like masses. It was shown that allowing for finite values of quark masses leads to considerable corrections to the string potential in comparison with calculations when the string ends are fixed (immobile quarks with infinitely heavy masses). The critical radius[a] in the string potential turns out to depend on the quark masses especially in the case of the asymmetric configurations when the string ties together light and heavy quarks [1].

Along with the critical radius of the interquark potential, an important parameter of the string model of hadrons is the temperature of deconfinement or critical temperature [6]. This prediction of the string model is directly compared with the lattice simulations in the framework of gauge theories [7]. In this connection the investigation of the quark mass contribution to the critical string temperature is of certain interest. It is this problem that will be treated in this paper.

Usually one believes that in string models the critical radius R_c of the potential and the critical temperature T_c are related. For example, in the Nambu-Goto string with fixed ends the relation $2\,R_c\,T_c = 1$ holds. As the critical radius R_c depends on the quark masses [2], one could expect that this will also be valid for the critical temperature T_c. However, this is not the case [8]. The deconfinement temperature in the Nambu-Goto string with point-like masses (quarks) at its ends turns out to be independent of the quark masses, this temperature being the same as that in the string with fixed ends.

The layout of the paper is the following. In Section 2, we will recall the definition of the critical temperature in string models. A new approach to calculate the Casimir energy at finite temperature in the Nambu-Goto string of finite length is then suggested in Section 3. The temperature dependence of this energy determines the deconfinement or critical temperature in the string model under consideration. We show in Section 4 that the critical temperature in theNambu-Goto string model with fixed ends and with point-like masses attached to the string ends is the same. Finally, in the conclusions,

[a] As is known [5], the potential generated by the string is not determined for any distances between quarks no sooner than at $R > R_c$, where R_c is the critical radius in the Nambu-Goto string with fixed ends which is determined by the string tension M_0^2: $R_c^2 = \pi\,(D-2)/(12\,M_0^2)$.

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the obtained results are briefly discussed.

2 Critical Temperature in String Models

The critical temperature (or the temperature of deconfinement) in string models is defined in the following way. Let $V(R,\,T)$ be the free energy of the string (or string potential) calculated at finite temperature T. The asymptotics of this energy at large distances is

$$V(R,\,T) \to \sigma(T)\,R, \quad R \to \infty, \tag{1}$$

where $\sigma(T)$ is an effective string tension depending on the temperature T. At the critical value of T ($T = T_c$), the string tension vanishes

$$\sigma(T_c) = \lim_{R\to\infty} R^{-1} V(R, T_c) = 0\,. \tag{2}$$

The string potential at finite temperature is directly calculated in the same way as at $T = 0$ [2]. As a result, one obtains the well known square-root expression

$$V(R,T,m) = M_0^2 R \sqrt{1 + \frac{2(D-2)}{M_0^2 R}\, E_C(R,T,m)}\,, \tag{3}$$

where M_0^2 is the string tension at zero temperature, i.e. $\sigma(T=0) = M_0^2$; m is the quark mass, $E_C(R,\,T,\,m)$ is the renormalized Casimir free energy for one transverse degree of freedom in the string model, and D is the dimension of the space-time. Usually one puts $D = 4$. Hence, determining the critical temperature requires the calculation of the Casimir free energy at finite temperature in the string model under investigation. For simplicity, we shall call the Casimir free energy briefly Casimir energy.

3 Casimir Energy at Finite Temperature

The Casimir energy in the Nambu-Goto string at finite temperature is given by [2]

$$E_C(R,T) = \frac{T}{2} \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{\infty} \ln(\Omega_n^2 + \omega_k^2)\,, \tag{4}$$

where Ω_n are the Matsubara frequencies $\Omega_n = 2\pi n T$, $n = 0, \pm 1, \ldots$. Here T is the temperature, and ω_k are the eigenfrequencies of the string determined

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by the boundary conditions for the string coordinates that, for fixed string ends (immobile quarks), assume the values

$$\omega_k = \frac{k\pi}{R}, \quad k = 1, 2, \ldots . \tag{5}$$

In the case of the string with masses at its ends, the frequencies ω_k are the positive roots of the following equation [2]

$$\tan(\omega\, R) = \frac{2\omega m}{\omega^2 - M_0^2} . \tag{6}$$

For simplicity the symmetric quark configuration is considered, i.e. $m_1 = m_2 = m$. Since the critical temperature is determined by the value of $E_C(R, T)$ in the limit $R \to \infty$ (see Eqs. (2) and (3)), it is convenient, in the case of string frequencies (5), to take this limit directly in Eq. (4) substituting the summation over k by integration

$$E_C^{\rm fixed}(R \to \infty, T) = \frac{T}{4} \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} dk \ln \left[\Omega_n^2 + \left(\frac{k\pi}{R} \right)^2 \right] = -\frac{\pi R T^2}{6} . \tag{7}$$

Here the divergent integral and the divergent sum over n are calculated by using the analytical regularization and the Riemann zeta function, respectively. Substituting (7) in (3) we find the critical temperature in the Nambu-Goto string with fixed ends

$$\frac{T_c}{M_0} = \sqrt{\frac{3}{\pi(D-2)}} . \tag{8}$$

However this method cannot be applied to string frequencies determined by Eq. (6). Investigating the double sum [9] in (5) without introducing the integration over dk is again based on the fact that frequencies ω_k are multiples of π/R.

Here we use a new method [10] for calculating the Casimir energy at finite temperature that works equally well both with frequencies (5) and with string frequencies determined by Eq. (6). The idea of the method is the following. At first we represent the renormalized Casimir energy at zero temperature in terms of the integral over string eigenfrequencies. In other words, we obtain

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the spectral representation

$$E_C(R, T=0) = \int\limits_0^\infty d\omega \, \mathcal{E}_C(R,\omega) \,. \tag{9}$$

Then we pass in a standard way [11,12] from integration to summation over the Matsubara frequencies $\Omega_n = 2\pi n T$, $n = 0, \pm 1, \pm 2, \ldots$. Practically this can be done by the following substitution in Eq. (9)

$$d\omega \to 2\pi T \, d\omega \sum_{n=0}^{\infty}{}' \delta(\omega - \Omega_n) \,, \tag{10}$$

with the result

$$E_C(R, T) = 2\pi T \sum_{n=0}^{\infty}{}' \mathcal{E}_C(R, \Omega_n) \,. \tag{11}$$

The prime in the summation symbol means that the term with $n = 0$ should be taken with a multiplier $1/2$.

It is worth noting here that the formal substitution (10) can lead to the free energy or to the internal energy of the quantum system under consideration. Therefore one has to be careful when applying this procedure [10].

The integral representation for the Casimir energy in the Nambu-Goto string with fixed ends is given by [2]

$$E_C^{\rm fixed} = \frac{1}{2\pi} \int\limits_0^\infty d\omega \, \ln\left(1 - \exp\left(-2\omega R\right)\right) = -\frac{R}{\pi} \int\limits_0^\infty \frac{\omega \, d\omega}{\exp\left(2\omega R\right) - 1} \,. \tag{12}$$

The last expression in this formula is obtained by integration by parts. Omitting the minus sign in (12), we see that the spectral density of energy in this formula is of Planckian form for the one-dimensional black body, the effective temperature of the string vacuum being equal to $(2R)^{-1}$. Applying the algorithm explained above, we find

$$E_C^{\rm fixed}(R, T) = -4\pi T^2 R \sum_{n=0}^{\infty}{}' \frac{n}{\exp\left(4\pi n R T\right) - 1} \,. \tag{13}$$

Integration by parts in (12) was required for obtaining the term with $n = 0$ in the sum (13) without divergencies. In order to overcome the stated difficulty in analogous calculations in statistical physics [11,12], the Casimir force is

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calculated at first and then the corresponding potential is recovered on this basis.

The sum in (13) can be evaluated in the two limiting cases of large and small temperatures. At large T, the main contribution in (13) is due to the term with $n = 0$:

$$E_C^{\rm fixed}(R, T \to \infty) = -\frac{T}{2}\,. \tag{14}$$

Using the Euler-Maclaurin formula for small T,

$$\sum_{n=0}^{\infty}{}' F(n) = \int_0^\infty F(x)dx - \frac{1}{12}F'(0) + \dots\,,$$

Eq. (13) reduces to the form

$$E_C^{\rm fixed}(R, T) = -\frac{\pi}{24\,R} - \frac{\pi R T^2}{6}\,. \tag{15}$$

Hence, unlike Eq. (7), we also preserve here the T-independent term $-\pi/24\,R$ which vanishes when $R \to \infty$. Proceeding from the physical consideration, it is clear that the string picture of quark confinement inside hadrons is applicable only at low temperatures. In string models the temperature scale is determined by the string tension $M_0 \sim 0.4$ GeV. Hence when finding the critical temperature in string models, the region of small temperatures should be considered [6]. After allowing all this, we have to substitute in (1)–(3) the expression for the Casimir energy at small T (Eq. (15)). After taking the limit $R \to \infty$ in (2), the contribution of the first term in (15) to the effective string tension vanishes and we obtain the critical temperature (8).

Equation (15) allows one to introduce a critical temperature for the string of a finite length $\bar{T}_c(R)$. To this end, $\lim_{R\to\infty}$ should be removed from definition (2). At this temperature, the effective tension of the string of finite length R vanishes. It is obvious that the string critical temperature defined in this way will be dependent on the string length R. By making use of Eqs. (2), (3), and (15), we obtain

$$\bar{\tau}_c^2(\rho) = \tau_c^2 - \frac{1}{4\rho^2}\,, \tag{16}$$

where the following dimensionless variables are introduced

$$\tau_c = T_c/M_0, \quad \rho = M_0 R\,. \tag{17}$$

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When $\rho \to \infty$, the critical temperature $\bar{\tau}_c(\rho)$, dependent on the string length ρ, tends to its limiting value τ_c from below, because the existence of longer strings requires a greater work for their splitting (phase transition) than it takes place in the case of short strings.

In another approach, the critical temperature for the Nambu-Goto string of finite length has been considered in Ref. [13]. The results obtained there probably imply that $\tau_c(\rho)$ tends to τ_c from above in the limit $\rho \to \infty$.

4 Critical Temperature for the Nambu-Goto String with Massive Ends

The method for calculating the Casimir energy at finite temperature presented in the preceding section can be directly applied to the Nambu-Goto string with massive ends. The integral representation of the Casimir energy at zero temperature in this string model is [2]

$$E_C(R, T=0, m) = \frac{1}{2\pi} \int_0^\infty d\omega \ln \left[1 - e^{-2\omega R} \left(\frac{\omega m - M_0^2}{\omega m + M_0^2} \right)^2 \right] , \tag{18}$$

where m is the quark mass. We consider, for simplicity, the quarks with equal masses (the Casimir energy and the string potential for different quark masses $m_1 \neq m_2$ have been investigated in Ref. [1]). Integrating by parts in (18), one obtains

$$E_C(R, T=0, m) = -\frac{R}{\pi} \int_0^\infty \frac{\omega\, d\omega}{g(\omega) - 1} \left(1 - \frac{2mM_0^2}{R(\omega^2 m^2 - M_0^4)} \right) , \tag{19}$$

where we have

$$g(\omega) = e^{2\omega R} \left(\frac{\omega m + M_0^2}{\omega m - M_0^2} \right)^2 . \tag{20}$$

Notice that now the spectral density of energy in (19) is not Planckian. Then, the Casimir energy at finite temperature is

$$E_C(R, T, m) = -{\sum_{n=0}^{\infty}}' \frac{4\pi T^2 R n}{g(2\pi n T) - 1} \left(1 - \frac{2mM_0^2}{R(4\pi^2 n^2 T^2 m^2 - M_0^4)} \right) , \tag{21}$$

where $g(x)$ is defined in (20). When $m \to \infty$ or $m \to 0$, Eq. (21) reduces to (13). It is important to note that the Casimir energies at finite temper-

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ature given by the sums (13) and (21) are free of any divergencies. It is a direct consequence of using the integral representations (12) and (18) for the *renormalized* Casimir energies at zero temperature.

In the limit $T \to \infty$, the Casimir energy (21) tends to the value

$$E_C(R, T \to \infty, m) = -\frac{T}{2}\left(1 + \frac{2m}{M_0^2 R}\right) . \tag{22}$$

At low temperature the estimation of the sum in (21) can be done by making again use of the Euler-MacLaurin formula, where the quantity $F'(0)$ should be calculated. Here, the function $F(x)$ is given by

$$F(x) = \frac{x(1-\eta x)^2}{e^{2Rx}(1+\eta x)^2 - (1-\eta x)^2}\left(1 - \frac{2\eta}{R(\eta^2 x^2 - 1)}\right) , \tag{23}$$

where η is the ratio m/M_0^2. Expanding $F(x)$ in a Taylor series gives

$$F(x) \simeq \frac{\left[1 - (R+2\eta)x + \mathcal{O}(x^2)\right]}{2(R+2\eta)}\left[1 + \frac{2\eta}{R}(1+\eta^2 x^2) + \mathcal{O}(x^4)\right] . \tag{24}$$

Then, it follows that

$$F'(0) = -\frac{1}{2}\left(1 + \frac{2\eta}{R}\right) \to -\frac{1}{2} \quad \text{when} \quad R \to \infty . \tag{25}$$

In the limit $R \to \infty$ the integral term in the Euler-McLaurin formula vanishes. Hence, the critical temperature in the model under investigation turns out to be the same as that in the Nambu-Goto string with fixed ends

$$\tau_c^2 = \left(\frac{T_c}{M_0}\right)^2 = \frac{3}{\pi(D-2)} . \tag{26}$$

If we consider again the critical temperature dependent on the string length (see the end of the preceding section), then this quantity proves to be dependent on the quark mass. Indeed, now we have, instead of (16),

$$\bar{\tau}_c^2(\rho, \mu) = \left(1 + \frac{2\mu}{\rho}\right)^{-1}\left(\tau_c^2 - \frac{I(q)}{4\rho^2}\right) , \tag{27}$$

where $\mu = m/M_0$, $\rho = M_0 R$, $q = M_0^2 R/m = \rho/\mu$, and where the function $I(q)$, generated by the integral term in the Euler-Mclaurin formula, is given

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by

$$I(q) = \frac{24}{\pi^2} \int_0^\infty dz \, \frac{z(z-q)}{e^{2z}(z+q)^2 - (z-q)^2} \frac{z^2 - q^2 - 2q}{z+q} \,. \tag{28}$$

When $m \to \infty$, the function $I(q)$ tends to 1. In order to obtain Eq. (16) from (27) in this limit, one has to send ρ to ∞ in the first multiplier in (27).

5 Conclusions

The method for calculating the Casimir energy at finite temperature proposed here enables us to find the critical temperature in the Nambu-Goto string with massive ends. Before it was not obvious that this temperature is the same as that in the string model with fixed string ends. Probably this is due to the fact that in string models, like in statistical models, the boundary effects appear to be unessential for implementing phase transitions.

Our consideration is restricted to the one-string approximation. Certainly, it is important to calculate the critical temperature for a gas of interacting strings with massive ends and to take in this way into account the processes of splitting and joining such strings which are followed by creation and annihilation of quark-antiquark pairs. Through this mechanism the quark masses will be involved in determining the deconfinement temperature in a more direct way. Unfortunately, the theory of interacting hadronic strings is still unknown. Therefore, new nonstandard methods should be developed for treating this problem.

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