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# DYNAMICAL FERMION MASSES UNDER THE INFLUENCE OF KALUZA-KLEIN FERMIONS IN RANDALL-SUNDRUM BACKGROUND

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The dynamical fermion mass generation on the D3-brane in the Randall-Sundrum space-time is discussed in a model with bulk fermions in interaction with fermions on the branes. It is found that the dynamical fermion masses are generated at the natural (R.-S.) radius of the compactified extra-dimensional space and may be made small compared with masses of the Kaluza-Klein modes which are of order TeV.

## 1 Introduction

In the 1920's an interesting idea came up assuming the existence of an extra-dimensional space which eventually compactifies leaving our 4-dimensional space-time as a real world [1]. A few years ago, a proposal [2] for the mass

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scale of the compactified space to be much smaller than the Planck scale gave a strong impact on the onset of studying phenomenological evidences of extra-dimensional effects [3]. There is a crucial problem, however, how such large extra dimensions are stabilized. Recently, Randall and Sundrum [4] (R.-S.) gave an alternative to the large extra dimension scenario. By introducing a specific curved bulk space they succeeded to get a mass scale much smaller than the Planck scale without relying on the fine-tuning.

In our present analysis we introduce bulk fermions in the R.-S. space-time [5] and see what effects could be observed. The bulk fermions interact with themselves as well as with fermions on the 4-dimensional branes through the exchange of the graviton and its Kaluza-Klein excited modes, or through the exchange of gauge bosons which may be assumed to exist in the bulk [5]. The interactions among fermions generated as a result of the exchange of all the Kaluza-Klein excited modes of the graviton or gauge bosons may be expressed as effective four-fermion interactions [3,6,7]. According to the four-fermion interactions we expect that the dynamical generation of fermion masses will take place. In the present communication we look for a possibility of the dynamical fermion mass generation under the influence of the bulk fermions through the effective four-fermion interactions in the R.-S. background. In the R.-S. background, fermion mass terms are forbidden by the $S^1/\mathbf{Z}_2$ symmetry. The possible source of fermion masses on the branes is two-fold, i.e. the dynamically generated fermion masses and masses of the Kaluza-Klein excited modes of the bulk fermions. The mass of the Kaluza-Klein excited modes is known to be of order of TeV [5].

We will review the dynamical fermion mass generation with bulk fermions in the *torus* compactified case [8] because most of the technical parts can be summarized in the torus (flat) extra dimension case. After that we proceed to the case of the R.-S. space-time in Section 3, the main part of this paper: It is found that the dynamical fermion masses are generated at the natural (R.-S.) radius of the compactified extra-dimensional space and may be made small compared with masses of the Kaluza-Klein modes which are of order of TeV because of the presence of the Randall-Sundrum warp factor.

## 2 Flat Extra Space

We assume an existence of 5-dimensional bulk fermions $\psi$ in interaction with fermions $L$ on the 4-dimensional brane. Effective interactions among these fermions can be given in the form of the four-fermion interaction. In fact it

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is known that the exchange of the Kaluza-Klein excited modes of the bulk graviton results in effective four-fermion interactions [3]. After the Fierz transformation on the four-fermion interactions we generate the transition-type interactions. Accordingly we start with the following Lagrangian for our model [8]

$$\mathcal{L}^{(5)} = \bar{\psi} i\gamma^M \partial_M \psi \, + \, [\, \bar{L} i\gamma^\mu \partial_\mu L \, + \, g^2 \bar{\psi}\gamma^M L \, \bar{L}\gamma_M \psi \,] \, \delta(x^4), \tag{1}$$

where $g$ is the coupling constant with mass dimension $-3/2$ and the index $M$ runs from 0 to 4 while the index $\mu$ runs from 0 to 3. Fermions $\psi$ and $L$ are assumed to consist of $N_f$ components.

After neglecting tensor interactions and introducing an auxiliary field $\sigma \sim \bar{\psi} L$, we compactify the bulk space on torus with radius $R$. Our 4-dimensional Lagrangian is rewritten as

$$\mathcal{L}^{(4)} = \bar{\Psi}(M + I i\partial\!\!\!/)\Psi \; - \; |\sigma|^2 \, , \tag{2}$$

where $\Psi^t \equiv (L \, , \, \psi_0 \, , \, \psi_1 \, , \, \psi_{-1} \, , \cdots)$ and

$$M \equiv \begin{pmatrix} 0 & m^* & m^* & m^* & \cdots \\ m & 0 & 0 & 0 & \cdots \\ m & 0 & \frac{1}{R} & 0 & \cdots \\ m & 0 & 0 & -\frac{1}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \qquad m = \frac{g\sigma}{\sqrt{2\pi R}}. \tag{3}$$

If $\sigma$ acquires a non-vanishing vacuum expectation value, we replace $\sigma$ in $m$ by its vacuum expectation value $\langle\sigma\rangle$, i.e. $m = Ng\,\langle\sigma\rangle$. The eigenvalues of the matrix $M$ determine the masses of 4-dimensional fermions. Obviously we find that the lightest eigenvalue is given by

$$\lambda_{\pm 0} = \pm \, |m| \quad \text{for} \quad |m| \ll 1/R \, . \tag{4}$$

Thus we conclude that there is a possibility of having the light fermion masses within our scheme being much smaller than the mass of the Kaluza-Klein modes of the bulk fermion. By performing the path integration for the fermion field $\Psi$, we find the effective potential for $\sigma$ in the leading order of the $1/N_f$ expansion:

$$V(\sigma) = |\sigma|^2 - \frac{1}{2\pi^2} \int_0^\Lambda dx \, x^3 \ln \left[ x^2 + |m|^2 (\pi x R) \coth(\pi x R) \right]$$

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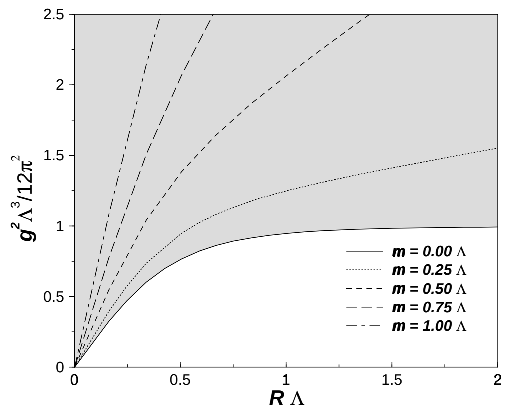


Figure 1. Critical radius as a function of $g$.

$$-\frac{1}{2\pi^2}\sum_{j=1}^{\infty}\int_0^{\Lambda} dx\, x^3 \ln\left[x^2+\left(\frac{j}{R}\right)^2\right]. \tag{5}$$

The gap equation to determine the vacuum expectation value $\langle|\sigma|\rangle$ of $|\sigma|$ reads

$$\frac{\partial V(\sigma)}{\partial|\sigma|} = 2|\sigma|\left\{1-\frac{g^2}{2\pi^2}\int_0^{\Lambda} dx\,\frac{x^3}{2x\tanh(\pi x R)+g^2|\sigma|^2}\right\} = 0. \tag{6}$$

By numerical integration of Eq. (6) we find that there exists a non-trivial solution for $|\sigma|$ for a suitable range of parameters $g$ and $R$, and that the solution corresponds to the true minimum of the effective potential. Accordingly, the fermion mass is generated dynamically. Here the auxiliary field $\sigma$ (or the composite field $\bar{L}\psi$) acquires a vacuum expectation value. Moreover it is easily confirmed that the phase transition associated with this symmetry breaking is of second order.

As shown in Eq. (4) the lowest fermion mass on the 4-dimensional brane is $m = Ng\langle|\sigma|\rangle$ where $\langle|\sigma|\rangle$ is determined by solving Eq. (6). The critical curve for $m = 0$ which represents the critical radius as a function of the coupling constant is shown in Fig. 1. If we assume $1/R \sim$ TeV, light ($<$ TeV) fermions are obtained in the region between the solid line and the dot-dashed line in Fig. 1 because it is natural that $R\Lambda \sim 1$. However, we have no idea how to justify $1/R \sim$ TeV and so we need to introduce a certain mechanism such as the Randall-Sundrum model.

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## 3 Warped Extra Space

People wonder how, in theories with large ($\gg M_{\rm pl}^{-1}$) extra dimensions, such large radii are stabilized. We have not yet come across with any satisfactory answer, while important results in many models depend essentially on the largeness of the radius (see the previous section). Randall and Sundrum [4] noticed that the existence of the brane leads to the curved bulk space and proposed the so-called Randall-Sundrum (R.-S.) model. Their model has *two* D3-branes on the $S_1/\mathbf{Z_2}$ orbifold fixed points and the ADS$_5$ between these branes:

$$ds^2 \equiv G_{MN}dx^M dx^N = e^{-2kb_0|y|}\eta_{\mu\nu}dx^\mu dx^\nu + b_0^2 dy^2, \tag{7}$$

where $k \sim M_{\rm pl}$ is the gravity scale and $b_0^{-1} \sim 10^{-1}k - 10^{-2}k$ is the compactification scale. One of the most important results is the warp factor $e^{-kb_0/2}$ which is the suppression of the K.-K. masses of the bulk fields [5].

### 3.1 Bulk Fermions in R.-S. Background

Following Chang *et al.* [5] we derive the mode expansion of the bulk fermion in R.-S. space-time such that

$$\psi(x,y) = \frac{e^{\frac{3}{2}kb_0|y|}}{\sqrt{b_0}} \sum_n \left[\psi_L^{(n)}(x)\xi(y) + \psi_R^{(n)}(x)\eta(y)\right], \tag{8}$$

$$\begin{cases} \xi(y) = \sqrt{\frac{kb_0}{1-e^{-\frac{1}{2}kb_0}}}\, e^{-\frac{1}{2}kb_0|\frac{1}{2}-y|} \sin\dfrac{m_n}{kb_0}(1-e^{kb_0|y|}) \\ \eta(y) = \sqrt{\frac{kb_0}{1-e^{-\frac{1}{2}kb_0}}}\, e^{-\frac{1}{2}kb_0|\frac{1}{2}-y|} \cos\dfrac{m_n}{kb_0}(1-e^{kb_0|y|}) \end{cases}, \tag{9}$$

where $m_n \equiv n\pi kb_0/(e^{\frac{1}{2}kb_0}-1)$ is $b_0$ times the mass of the $n$-th K.-K. mode. With this expansion the kinetic terms of the K.-K. modes become

$$\mathcal{L}_{\rm bulk}^{(4)} = \int dy\, E\bar{\psi}i\gamma^M D_M\psi = \sum_n \left[\bar{\psi}^{(n)}i\partial\!\!\!/\psi^{(n)} - \frac{m_n}{b_0}\bar{\psi}^{(n)}\psi^{(n)}\right]. \tag{10}$$

We can now apply previous results of our model with bulk fermions in the R.-S. background.

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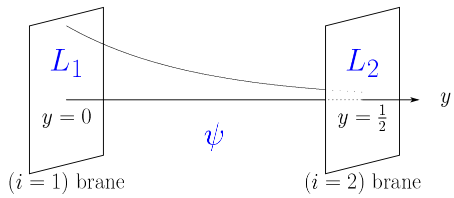


Figure 2. Four-fermion interaction model in R.-S. background.

### 3.2 Four-Fermion Interaction Model in R.-S. Background

When applying the previous torus-compactified model (1) to the case in the R.-S. background, we get

$$E_{\bar{M}}{}^{M} = \begin{pmatrix} e^{kb_0|y|}\eta_{\bar{\mu}}{}^{\mu} & 0 \\ 0 & b_0^{-1} \end{pmatrix}, \tag{11}$$

where $E_{\bar{M}}{}^{M}$ is the vielbein whose square becomes the metric $G_{MN}$. We introduce the bulk fermion $\psi$ which propagates in the whole five-dimensional space as in the torus case, while *two* brane fermions $L_i$ $(i=1,2)$ propagate on an $i$-th "brane".

We start with the Lagrangian in five dimensions,

$$\mathcal{L}^{(5)} = E\bar{\psi}i\gamma^M D_M\psi + E_{(1)}\left[\bar{L}_1 i\gamma^\mu\partial_\mu L_1 + \frac{g_1^2}{N_f}\bar{\psi}\gamma^M L_1\bar{L}_1\gamma_M\psi\right]\delta(x^4)$$
$$+E_{(2)}\left[\bar{L}_2 i\gamma^\mu\partial_\mu L_2 + \frac{g_2^2}{N_f}\bar{\psi}\gamma^M L_2\bar{L}_2\gamma_M\psi\right]\delta(x^4-\tfrac{1}{2}), \tag{12}$$

where $x^M=(x^\mu, x^4=y)$, $\mu=(0,1,2,3)$, $g_i$'s are the four-fermion couplings ($[g_i]=[-3/2]$), and $N_f$ is the number of components of $\psi$ and the $L_i$'s. Note that the bulk mass term is removed by the $S^1/\mathbf{Z_2}$ projection. Because Lorentz covariance should be preserved, we neglect the tensor (vector) interactions and obtain

$$\mathcal{L}^{(5)} = E\bar{\psi}i\gamma^M D_M\psi + \left[\bar{L}_1 i\gamma^{\bar{\mu}}\partial_{\bar{\mu}} L_1 + \frac{g_1^2}{N_f}\bar{\psi}\gamma_5 L_1\bar{L}_1\gamma_5\psi\right]\delta(x^4)$$
$$+e^{-2kb_0}\left[e^{\frac{1}{2}kb_0}\bar{L}_2 i\gamma^{\bar{\mu}}\partial_{\bar{\mu}} L_2 + \frac{g_2^2}{N_f}\bar{\psi}\gamma_5 L_2\bar{L}_2\gamma_5\psi\right]\delta(x^4-\tfrac{1}{2}). \tag{13}$$

After chiral rotation $\psi\to e^{i\pi\gamma^5/4}\psi$ and $L_i\to e^{i\pi\gamma^5/4}L_i$, we introduce auxiliary

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fields $\sigma_i$ to obtain

$$\mathcal{L}^{(5)} = E\bar{\psi}i\gamma^M\partial_M\psi + \left[\bar{L}_1 i\partial\!\!\!/ L_1 - N_f\,|\sigma_1|^2 + (g_1\sigma_1\bar{\psi}L_1 + h.c.)\right]\delta(x^4)$$
$$+e^{-2kb_0}\left[e^{\frac{1}{2}kb_0}\bar{L}_2 i\partial\!\!\!/ L_2 - N_f\,|\sigma_2|^2 + (g_2\sigma_2\bar{\psi}L_2 + h.c.)\right]\delta(x^4 - \tfrac{1}{2}). \quad (14)$$

Now we substitute the mode expansion of $\psi$ of Eq. (8) into the Lagrangian and integrate it over the extra-dimension. Taking $e^{-3kb_0/4}L_2 \to L_2$ and $e^{-kb_0}\sigma_2 \to \sigma_2$, the Lagrangian in four dimensions becomes

$$\mathcal{L}^{(4)} \equiv \int dy\,\mathcal{L}^{(5)}$$
$$= \sum_n \left[\bar{\psi}^{(n)} i\partial\!\!\!/ \psi^{(n)} - \frac{n}{R}\bar{\psi}^{(n)}\psi^{(n)}\right] + \bar{L}_1 i\partial\!\!\!/ L_1 + \bar{L}_2 i\partial\!\!\!/ L_2 - N_f(|\sigma_1|^2 + |\sigma_2|^2)$$
$$+\sum_n \left(\mu_1\bar{\psi}_R^{(n)}L_1 + h.c.\right) + \sum_n \left((-1)^n\mu_2\bar{\psi}_R^{(n)}L_2 + h.c.\right), \quad (15)$$

where

$$\mu_1 \equiv \frac{g_1\sigma_1}{\sqrt{b_0}}, \quad \mu_2 \equiv \frac{g_2\sigma_2}{\sqrt{b_0}}, \quad R \equiv \frac{b_0}{m_1} = \frac{e^{\frac{1}{2}kb_0} - 1}{\pi k} \sim (\mathrm{TeV})^{-1}. \quad (16)$$

Here we note that the mass of the first K.-K. excited mode $1/R$ is of the order of TeV without fine-tuning because of the warp factor in the R.-S. metric. This is the only dependence on the factor.

After integrating out all fermionic degrees of freedom, we obtain the effective potential for $\sigma_i(\mu_i)$ as follows:

$$\bar{V}(\mu_1/\Lambda, \mu_2/\Lambda)$$
$$= [\,V(\mu_1/\Lambda, \mu_2/\Lambda) - V(0,0)\,]/\Lambda^4$$
$$= 2\pi R\Lambda\left[\frac{(\mu_1/\Lambda)^2}{g_1^2\Lambda^3} + \frac{(\mu_2/\Lambda)^2}{g_2^2\Lambda^3}\right] - \frac{1}{2\pi^2}\int_0^1 dz\,z^3\ln\Bigg\{1 +$$
$$\frac{\pi R\Lambda}{z}\left[\left(\frac{\mu_1}{\Lambda}\right)^2 + \left(\frac{\mu_2}{\Lambda}\right)^2\right]\cot(\pi z R\Lambda) + \frac{\pi^2}{2}(R\Lambda)^2\left(\frac{\mu_1}{\Lambda}\right)^2\left(\frac{\mu_2}{\Lambda}\right)^2\Bigg\}. \quad (17)$$

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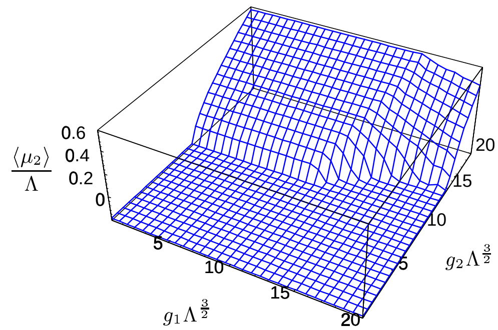


Figure 3. $\langle\mu_2\rangle$ as a function of $g_1, g_2$.

### *3.3 Behavior of the Vacuum* $\langle\mu_i\rangle$

The behavior of the vacuum is determined by solving the gap equations ($i, j = 1, 2;\ i \neq j$):

$$\begin{aligned}\frac{\partial\bar{V}}{\partial(\mu_i/\Lambda)} &= R\Lambda\frac{\mu_i}{\Lambda}\left\{\frac{4\pi}{g_i^2\Lambda^3} - \frac{1}{2\pi}\int_0^1 dz\, z^3\right. \\ &\times\left.\frac{\pi R\Lambda\mu_j^2\tanh(\pi z R\Lambda) + 2}{z\left[1+\frac{\pi^2}{2}(R\Lambda)^2\left(\frac{\mu_1}{\Lambda}\right)\left(\frac{\mu_2}{\Lambda}\right)\right]\tanh(\pi z R\Lambda) + \pi R\Lambda\left[\left(\frac{\mu_1}{\Lambda}\right)+\left(\frac{\mu_2}{\Lambda}\right)\right]}\right\} \\ &= 0. \end{aligned} \tag{18}$$

The system has a second-order phase transition. As shown in Fig. 3, $\langle\mu_2\rangle$ is a function of $g_1$ and $g_2$ for $R\Lambda = 1$. If $g_i \gtrsim g_{i,\text{critical}}$, we see that $\langle\mu_i\rangle \gtrsim 0$. The phase structure of this system is summarized in Fig. 4.

### *3.4 Effective Theory on* $y = 1/2$ *Brane*

Now we realize that $\mu_1$ and $\mu_2$ can have a non-zero vacuum expectation value and that the Lagrangian (15) has the mixing term $\bar{\psi}_R L_i + \text{h.c.}$. The physics on the $i = 2$ brane is examined by integrating out the invisible field $L_1$. Setting $\sqrt{2}\psi_L^{(n)} \equiv N^{(n)} - M^{(n)}$ and $\sqrt{2}\psi_R^{(n)} \equiv N^{(n)} + M^{(n)}$ for $n \neq 0$, the effective Lagrangian on the $y = 1/2$ brane is obtained as follows:

$$\mathcal{L}_{\text{eff}}^{(4)} = \bar{\Theta}_2[\,i\partial\!\!\!/ + M_2 + |\mu_1|^2 P\,]\Theta_2, \tag{19}$$

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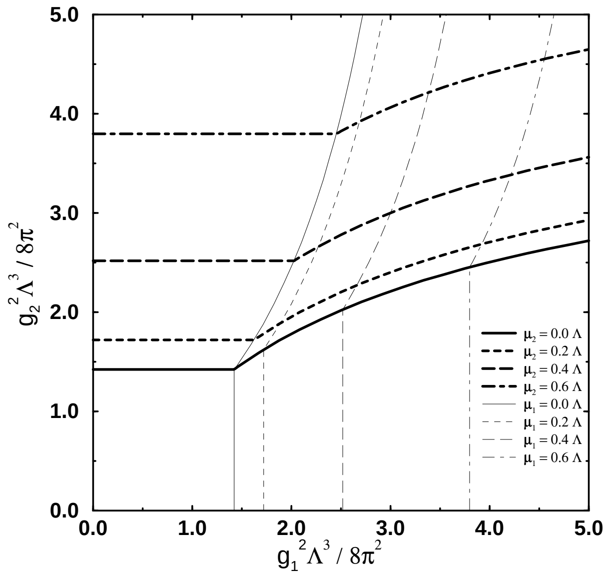


Figure 4. The phase structure of the vacuum $\langle\mu_2\rangle$.

where $\Theta_2^T \equiv \left(L_2\,,\,\psi_R^{(0)}\,,\,N^{(1)}\,,\,M^{(1)}\,,\ldots\right)$ and

$$M_2 \equiv \begin{pmatrix} 0 & \mu_2^* & -\mu_2^* & -\mu_2^* & \cdots \\ \mu_2 & 0 & 0 & 0 & \cdots \\ -\mu_2 & 0 & \frac{1}{R} & 0 & \cdots \\ -\mu_2 & 0 & 0 & -\frac{1}{R} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},\; P \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & (i\partial\!\!\!/)^{-1} & (i\partial\!\!\!/)^{-1} & (i\partial\!\!\!/)^{-1} & \cdots \\ 0 & (i\partial\!\!\!/)^{-1} & (i\partial\!\!\!/)^{-1} & (i\partial\!\!\!/)^{-1} & \cdots \\ 0 & (i\partial\!\!\!/)^{-1} & (i\partial\!\!\!/)^{-1} & (i\partial\!\!\!/)^{-1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (20)$$

We have seen in the previous subsection that the dynamical fermion mass generation with $\psi$ is a second-order phase transition, and we find the parameter region of $(g_1, g_2)$ for $\langle\mu_2\rangle \lesssim \Lambda$, $1/R \sim$ TeV, $\langle\mu_1\rangle = 0$. Thus in Eq. (19) we have a possibility to get a light Dirac fermion on the $i = 2$ brane.

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## 4 Conclusion

In our model the dynamically generated fermion mass [9] is much smaller than the Planck scale because of the presence of the mass scale $1/R \sim$ TeV generated by the Randall-Sundrum warp factor. Furthermore, in spite of the presence of the mass scale $1/R \sim$ TeV in the theory, the fermion masses on the four-dimensional brane can be made smaller than this scale as a consequence of the interaction among the bulk and brane fermions: the mixing of the brane fermions with the bulk fermions does not lead to the lightest fermion masses of order $1/R$ and the dynamically generated fermion masses are not of order $1/R$. This result is obtained because the dynamical fermion masses generated in the second-order phase transition are small irrespective of $1/R$ near the critical radius. In our model the possibility of having low mass fermions resulted from the dynamical origin. This mechanism is quite different from the ones in other approaches in which low mass fermions are expected to show up as a result of the kinematical origins [10-14].

As an outlook we want to point out four items. The first item is whether the K.-K. gauge boson (or graviton) exchange induces suitable effective four-fermion interactions. The second is to understand the physics with $\langle\mu_1\rangle \neq 0$ and the third is how to stabilize the radius $b_0 = 10 \sim 100k^{-1}$ with the pressure of the bulk fermion $\psi$ [15-17]. The last is the extension of our model to electroweak symmetry breaking [6,7,18,19].

## Acknowledgments

This paper is dedicated to the 60th birthday of Professor Hagen Kleinert who is a good old friend of one of the authors (T.M.). The authors would like to thank Hironori Miguchi, Koichi Yoshioka (Kyoto U.), Masahiro Yamaguchi (Tohoku U.), and Hiroaki Nakano (Niigata U.) for fruitful discussions and correspondence. They are also indebted to Tak Morozumi for useful comments. The present work was financially supported by the Monbusho Grant, Grant-in-Aid for Scientific Research (C) with contract number 11640280.

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