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# PHASE TRANSITION IN THE RANDOM ANISOTROPY MODEL

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The influence of a local anisotropy of random orientation on a ferromagnetic phase transition is studied for two cases of anisotropy axis distribution. To this end a model of a random anisotropy magnet is analyzed by means of the field theoretical renormalization group approach in two-loop approximation refined by a resummation of the asymptotic series. The one-loop result of Aharony indicating the absence of a second-order phase transition for an isotropic distribution of random anisotropy axis at space dimension $d < 4$ is corroborated. For a cubic distribution the accessible stable fixed point leads to disordered Ising-like critical exponents.

## 1 Introduction

Modern understanding of universal properties of matter in the vicinity of critical points is mainly due to the application of renormalization group (RG) ideas [1]. Applied to the problems of condensed matter physics in the early

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1970s, the RG technique proved to be a powerful tool to study critical phenomena. For example, expressions for critical exponents governing the magnetic phase transition in regular systems are known by now with record accuracy both for isotropic [2] [$O(m)$ symmetrical] and cubic [3] magnets. The RG approach also sheds light on the influence of structural disorder on ferromagnetism. In the present article we will apply the field theoretical RG approach to study peculiarities of magnetic behavior influenced by disorder in a form of random anisotropy axis [4]. It is a special pleasure for us to dedicate this paper to Prof. Hagen Kleinert on the occasion of his 60th anniversary. His contribution to the field is hard to be overestimated.

Although an influence of a weak quenched structural disorder on universal properties of a ferromagnetic phase transition has already been a problem of intensive study for several decades, there remains a number of unsettled questions. Here, one should distinguish between random site, random field and random anisotropy magnet. A weak quenched disorder preserves the second-order phase transition in three-dimensional ($d = 3$) random site magnets [5] but can destroy this transition in random field systems [6] for $d < 4$. The situation for the random anisotropy magnets is not so clear.

Typical examples of random-anisotropy magnets are amorphous rare-earth - transition metal alloys. Some of these systems order magnetically and for the description of the ordered structure it has been proposed [4] to consider a regular lattice of magnetic ions, each of them being subjected to a local anisotropy of random orientation. The Hamiltonian of this random anisotropy model (RAM) reads [4]

$$\mathcal{H} = -\sum_{\mathbf{R},\mathbf{R}'} J_{\mathbf{R},\mathbf{R}'} \vec{S}_{\mathbf{R}} \vec{S}_{\mathbf{R}'} - D_0 \sum_{\mathbf{R}} (\hat{x}_{\mathbf{R}} \vec{S}_{\mathbf{R}})^2, \qquad (1)$$

where $\vec{S}_{\mathbf{R}}$ is an $m$-component vector on a lattice site $\mathbf{R}$, $J_{\mathbf{R},\mathbf{R}'}$ is an exchange interaction, $D_0$ is an anisotropy strength, and $\hat{x}_{\mathbf{R}}$ is a unit vector pointing in the local (quenched) random direction of an uniaxial anisotropy.

The model has been investigated by a variety of techniques including mean-field theory [7], computer simulations [8], $1/m$-expansion [9], renormalization group $\varepsilon$-expansion [10–12]. The limit case of an infinite anisotropy has been subject to a detailed study as well [13,14]. However the nature of the low-temperature phase in RAM is not completely clear up to now, although several low-temperature phases were discussed like ferromagnetic ordering [7,8], spin-glass phase [8,9], and quasi long-range ordering [15].

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The nature of ordering is connected with the distribution of the random variables $\hat{x}_{\mathbf{R}}$ in Eq. (1). For an isotropic distribution arguments similar to those applied by Imry and Ma [16] for a random-field Ising model bring about the absence of ferromagnetic order for space dimensions $d < 4$ [12,17], whereas anisotropic distributions may lead to a ferromagnetic order [18].

Application of the Wilson RG technique to RAM with the isotropic distribution of a local anisotropy axis suggests [10] the possibility of "runaway" solutions of the recursion equations. Such a behavior has been interpreted as a smeared transition. However this result was obtained in first order of the $\varepsilon$-expansion and remains to be confirmed also in higher orders.

Here, we will report results obtained by means of the field theoretical RG technique in two-loop approximation refined by a resummation of the resulting asymptotic series. We will consider two cases of distribution of the random anisotropy axis and show that a ferromagnetic second-order phase transition takes place only when the distribution is non-isotropic. Moreover we will show that the RAM provides another example of a disordered model, where the only possible new critical behavior is of "random Ising" type, similar to the site-diluted magnets [5]. More detailed results can be found in Refs. [19,20].

## 2 Isotropic Case

In order to deal with quenched disorder, one way to obtain the effective Hamiltonian of a RAM is to make use of the replica trick. For a given configuration of quenched random variables $\hat{x}_{\mathbf{R}}$ in Eq. (1) the partition function may then be written in the form of a functional integral of a Gibbs distribution depending on $\hat{x}_{\mathbf{R}}$. To average over configurations, one should complete the model by choosing a certain distribution of $\hat{x}_{\mathbf{R}}$. We will analyze two cases: first the isotropic case, where the random vector $\hat{x}$ points with equal probability in any direction of the $m$-dimensional hyperspace, and second the cubic case, where $\hat{x}$ lies along the edges of the $m$-dimensional hypercube. Other distributions may be considered as well. In the first case the distribution function reads:

$$p(\hat{x}) \equiv \left( \int d^m \hat{x} \right)^{-1} = \frac{\Gamma(m/2)}{2\pi^{m/2}}. \tag{2}$$

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Following the above described program, one ends up with the replica $n \to 0$ limit of the effective Hamiltonian [10]:

$$\mathcal{H}_{\text{eff}} = -\int d^dR \left\{ \frac{1}{2}\left[{\mu_0}^2|\vec{\varphi}|^2 + |\vec{\nabla}\vec{\varphi}|^2\right] + u_0|\vec{\varphi}|^4 + v_0 \sum_{\alpha=1}^{n} |\vec{\phi}^\alpha|^4 \right.$$
$$\left. + w_0 \sum_{\alpha,\beta=1}^{n} \sum_{i,j=1}^{m} \phi_i^\alpha \phi_j^\alpha \phi_i^\beta \phi_j^\beta \right\}, \tag{3}$$

where ${\mu_0}^2$ and $u_0$, $v_0$, $w_0$ are defined by $D_0$ and familiar bare couplings of an $m$-vector model, and $\vec{\phi}^\alpha \equiv \vec{\phi}^\alpha_{\mathbf{R}}$ is an $m$-dimensional vector, $|\vec{\varphi}|^2 = \sum_{\alpha=1}^{n} |\vec{\phi}^\alpha|^2$. The bare couplings are restricted by $u_0 > 0$, $v_0 > 0$, $w_0 < 0$. Furthermore, values of $u_0$ and $w_0$ are related to appropriate cumulants of the distribution function (2) and their ratio equals $w_0/u_0 = -m$. Note that the symmetry of the $u_0$ and $v_0$ terms corresponds to the random site $m$-vector model [21]. However the $u_0$-term has an opposite sign.

In order to study long-distance properties of the Hamiltonian (3), we use the field theoretical RG approach [1]. Here the critical point of a system corresponds to a stable fixed point (FP) of the RG transformation. We apply the massive field theory renormalization scheme [22] performing renormalization at fixed space dimension $d$ and zero external momenta. In two-loop approximation we get [19] expressions for the RG functions in form of asymptotic series in renormalized couplings $u, v, w$.

As it was mentioned in the introduction, the only known RG results for RAM with isotropic distribution of the local anisotropy axis so far are those obtained in first order in $\varepsilon$ [10]. In total one obtains eight fixed points. All FPs with $u > 0, v > 0, w < 0$ appear to be unstable for $\varepsilon > 0$ except of the "polymer" $O(n = 0)$ FP III which is stable for all $m$ (see Fig. 1). However the presence of a stable FP is not a sufficient condition for a second-order phase transition. In order to be physically relevant, the FP should be accessible from the initial values of couplings. This is not the case for the location of FPs shown in Fig. 1. Indeed starting from the region of physical initial conditions (denoted by the cross in Fig. 1) in the plane of $v = 0$ one would have to cross the separatrix joining the unstable FPs I and VI. This is not possible and so one never reaches the stable FP III. As far as both FPs I and VI are strongly unstable with respect to $v$, FP III is not accessible for arbitrary positive $v$ either. Finally, the runaway solutions of the RG equations show that the second-order phase transition is absent in the model. The main question of

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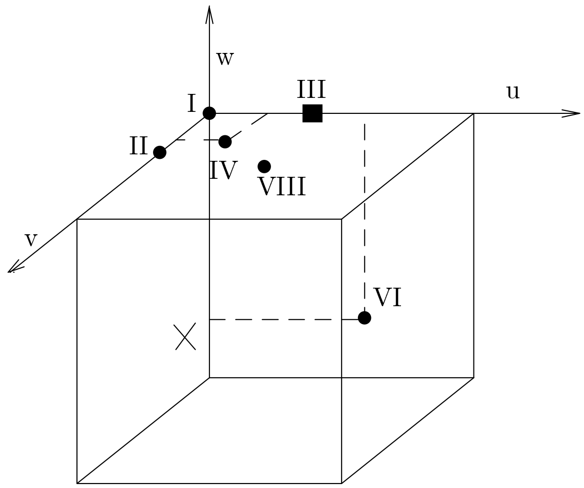


Figure 1. Fixed points of the RAM with isotropic distribution of a local anisotropy axis. The fixed points located in the octant $u > 0, v > 0, w < 0$ are shown. The filled box shows the stable fixed point, the cross denotes typical initial values of couplings.

Table 1. Resummed values of the fixed points and critical exponents for the isotropic case in two-loop approximation for $d = 3$. We absorb the value of a one-loop integral into the normalization of the couplings.

| FP | $m$ | $u^*$ | $v^*$ | $w^*$ | $\nu$ | $\eta$ |
|---|---|---|---|---|---|---|
| I | $\forall m$ | 0 | 0 | 0 | | |
| II | 2 | 0 | 0.9107 | 0 | 0.663 | 0.027 |
| | 3 | 0 | 0.8102 | 0 | 0.693 | 0.027 |
| | 4 | 0 | 0.7275 | 0 | 0.724 | 0.027 |
| III | $\forall m$ | 1.1857 | 0 | 0 | 0.590 | 0.023 |
| IV | 2 | -0.0322 | 0.9454 | 0 | 0.668 | 0.027 |
| | 3 | 0.1733 | 0.6460 | 0 | 0.659 | 0.027 |
| | 4 | 0.2867 | 0.4851 | 0 | 0.653 | 0.028 |
| VI | 2 | 1.4650 | 0 | -1.6278 | 0.449 | -0.028 |
| VIII | 2 | 0.7517 | 0.7072 | -0.3984 | 0.626 | 0.031 |
| | 3 | 0.8031 | 0.5463 | -0.3305 | 0.620 | 0.029 |
| | 4 | 0.8349 | 0.4545 | -0.2888 | 0.617 | 0.029 |

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interest is whether the above described picture of runaway solutions is not an artifact of the $\varepsilon$-expansion. To check this, we use a more refined analysis of the FPs and their stability, considering the series for RG functions directly at $d = 3$ [22]. It is known that series of this type are at best asymptotic and a resummation procedure has to be applied to obtain reliable data on their basis. We make use of Padé-Borel resummation techniques [23], first writing the RG functions as resolvent series [24] in one auxiliary variable and then performing the resummation. Numerical values of the FPs are given in Table 1. Resummed two-loop results qualitatively confirm the picture obtained in first-order $\varepsilon$-expansion: the stability of the FPs does not change after the resummation. This supports the conjecture of Aharony [10] that an accessible stable FP for the RAM with isotropic distribution of the local anisotropy axis is absent. In the table, we list values of correlation length and pair correlation function critical exponents $\nu$ and $\eta$ which are resummed in a similar way. As they are calculated in unstable FPs, they have rather to be considered as effective ones.

## 3 Cubic Case

Let us now consider the second example of anisotropy axis distribution, when the vector $\hat{x}_{\mathbf{R}}$ in Eq. (1) points only along one of the $2m$ directions of axes $\hat{k}_i$ of a cubic lattice:

$$p(\hat{x}) = \frac{1}{2m}\sum_{i=1}^{m}[\delta^{(m)}(\hat{x}-\hat{k}_i)+\delta^{(m)}(\hat{x}+\hat{k}_i)]. \tag{4}$$

The *rationale* for such a choice is to mimic the situation when an amorphous magnet still "remembers" the initial (cubic) lattice structure. Repeating the procedure described in the previous section, one ends up with the following effective Hamiltonian which is of interest in the limit $n \to 0$ [10]:

$$\mathcal{H}_{\text{eff}} = -\int d^dR \left\{ \frac{1}{2}\left[{\mu_0}^2|\vec{\varphi}|^2 + |\vec{\nabla}\vec{\varphi}|^2\right] + u_0|\vec{\varphi}|^4 + v_0\sum_{\alpha=1}^{n}|\vec{\phi}^{\alpha}|^4 \right.$$
$$\left. + w_0\sum_{i=1}^{m}\sum_{\alpha,\beta=1}^{n}{\phi_i^{\alpha}}^2{\phi_i^{\beta}}^2 + y_0\sum_{i=1}^{m}\sum_{\alpha=1}^{n}{\phi_i^{\alpha}}^4 \right\}. \tag{5}$$

Here, the bare couplings are $u_0 > 0$, $v_0 > 0$, $w_0 < 0$. The $y_0$ term is generated when the RG transformation is applied and may be of either sign.

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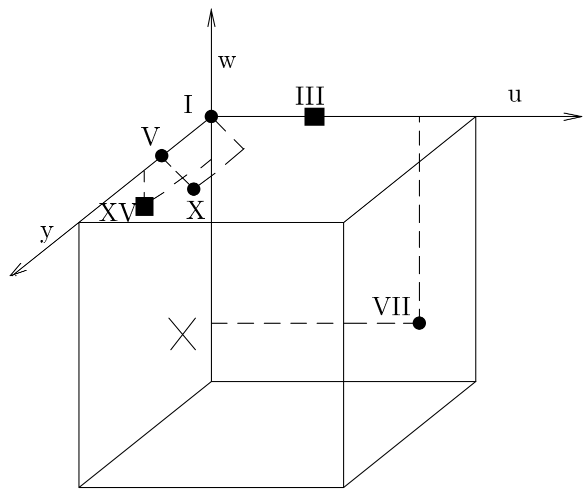


Figure 2. Fixed points of the RAM with distribution of a local anisotropy axis along hypercube axes for $v = 0$. The only fixed points located in the region $u > 0, w < 0$ are shown. Filled boxes show the stable fixed points, the cross denotes typical initial values of the couplings.

The symmetry of the $w_0$ terms differs in Eqs. (3) and (5). Furthermore, values of $w_0$ and $u_0$ differ for Hamiltonians (3) and (5), but their ratio equals $-m$ again.

We apply the massive field theory renormalization scheme [22] and get the RG functions in two-loop approximation [20]. As in the previous case we reproduce the first-order $\varepsilon$-results [10]. Now one gets 14 FPs. However, in first order of the $\varepsilon$-expansion all FPs with $u > 0, v > 0, w < 0$ appear to be unstable for $\varepsilon > 0$, except of the "polymer" $O(n = 0)$ FP III which is stable for all $m$ but not accessible (see Fig. 2). Now the account of the $\varepsilon^2$-terms qualitatively changes the picture. Indeed, the system of equations for the FPs appears to be degenerated at the one-loop level. As known from other cases in two-loop order, this leads to the appearance of a new FP which is stable and is expressed by a $\sqrt{\varepsilon}$ series [21]. The possibility of such a scenario was predicted already in Refs. [18]. However it remained unclear whether there exist any other accessible stable FPs.

Applying Padé-Borel resummation, we get 16 FPs. Values of the FPs with $u^* > 0, v^* > 0, w^* < 0$ are listed in Table 2. The last FP XV in Table 2 corresponds to the stable FP of the $\sqrt{\varepsilon}$-expansion. It has coordinates with $u^* = v^* = 0$, $w^* < 0$ and $y^* > 0$ and is accessible from the typical initial

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Table 2. Resummed values of the FPs and critical exponents for cubic distribution in two-loop approximation for $d = 3$. We absorb the value of a one-loop integral into the normalization of the couplings.

| FP | $m$ | $u^*$ | $v^*$ | $w^*$ | $y^*$ | $\nu$ | $\eta$ |
|---|---|---|---|---|---|---|---|
| I | $\forall m$ | 0 | 0 | 0 | 0 | 1/2 | 0 |
| II | 2 | 0 | 0.9107 | 0 | 0 | 0.663 | 0.027 |
| | 3 | 0 | 0.8102 | 0 | 0 | 0.693 | 0.027 |
| | 4 | 0 | 0.7275 | 0 | 0 | 0.720 | 0.026 |
| III | $\forall m$ | 1.1857 | 0 | 0 | 0 | 0.590 | 0.023 |
| V | $\forall m$ | 0 | 0 | 0 | 1.0339 | 0.628 | 0.026 |
| VI | 3 | 0.1733 | 0.6460 | 0 | 0 | 0.659 | 0.027 |
| | 4 | 0.2867 | 0.4851 | 0 | 0 | 0.653 | 0.027 |
| VII | $\forall m$ | 2.1112 | 0 | -2.1112 | 0 | 1/2 | 0 |
| VIII | 2 | 0 | 1.5508 | 0 | -1.0339 | 0.628 | 0.026 |
| | 3 | 0 | 0.8393 | 0 | -0.0485 | 0.693 | 0.027 |
| | 4 | 0 | 0.5259 | 0 | 0.3624 | 0.709 | 0.026 |
| IX | 3 | 0.1695 | 0.7096 | 0 | -0.1022 | 0.659 | 0.027 |
| | 4 | 0.2751 | 0.4190 | 0 | 0.1432 | 0.653 | 0.027 |
| X | $\forall m$ | 0.6678 | 0 | -0.6678 | 1.0339 | 0.628 | 0.026 |
| XV | $\forall m$ | 0 | 0 | -0.4401 | 1.5933 | 0.676 | 0.031 |

values of couplings (shown by a cross in Fig. 2). Applying the resummation procedure, we have not found any other stable FPs in the region of interest. The effective Hamiltonian (5) at $u = v = 0$ in the replica limit $n \to 0$ reduces to a product of $m$ effective Hamiltonians of a weakly diluted quenched random site Ising model. This means that for any value of $m > 1$ the system is characterized by the same set of critical exponents as those of a weakly diluted random site quenched Ising model. In Table 2, we give values of critical exponents in the other FPs as well: if the flows from initial values of couplings pass near these FPs, one may observe an effective critical behavior governed by these critical exponents.

## 4 Conclusions

We applied the field theoretical RG approach to analyze the critical behavior of a model of random anisotropy magnets with isotropic and cubic distribu-

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tions of a local anisotropy axis. The origin of a low-temperature phase in this model is not completely clear. General arguments based on an estimate of the energy for formation of magnetic domains [16] lead to the conclusion that for $d < 4$ a ferromagnetic order is absent [12,17]. However, these arguments do not take into account the entropy which may be important for disordered systems [14]. Furthermore, they do not apply for anisotropic distributions of the random axis [18].

In the RG analysis the absence of a ferromagnetic second-order phase transition corresponds to the lack of a stable FP of the RG transformation. However in the case of RAM with isotropic distribution of a local anisotropy axis the scenario differs. Our two-loop calculation leads to a $O(n = 0)$ symmetric FP which is stable for any value of $m$ for both isotropic and cubic distributions of a random anisotropy axis. Note that this FP is not accessible from the initial values of the couplings. We checked the location of the FPs up to second order in the $\varepsilon$-expansion and by means of a fixed $d = 3$ technique refined by Padé-Borel resummation.

In the case of isotropic distribution of a random anisotropy axis our analysis supports the conjecture of Aharony [10] based on results linear in $\varepsilon$ about runaway solutions of the RG equations. For the cubic distribution we get two stable FPs. One of them (FP III in Fig. 2) is not accessible as in the isotropic case. But the disordered Ising-like FP (FP XV in Fig. 2) may be reached from the initial values of couplings. Applying the resummation procedure we have not found any other stable FPs in the region of interest. This means that RAM with cubic distributions of a random anisotropy axis is governed by a set of critical exponents of a weakly site diluted quenched Ising model [21,25].

To conclude we want to attract attention to a certain similarity in the critical behavior of both random-site [21] and random-anisotropy [4] quenched magnets: if there appears a *new critical behavior* at all, it *is always governed by critical exponents of site-diluted Ising type*. The above calculations of a "phase diagram" of RAM are based on two-loop expansions improved by a resummation technique. Once the qualitative picture becomes clear, there is no need to go into higher orders of a perturbation theory as far as the critical exponents of the site-diluted Ising model are known by now with high accuracy [25].

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