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RANDOM PATHS AND SURFACES WITH RIGIDITY

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We discuss discretized models of random paths and surfaces with particular emphasis on a rigorous analysis of the influence of rigidity terms in the action upon the continuum properties.

1 Introduction

In the present article we study a class of models of fluctuating paths and surfaces, whose statistical weight depends on geometric properties of the fluctuating object. In particular, a rigidity dependence may be introduced in the form of curvature dependent terms in the action functional. Our main purpose is to outline results from a rigorous analysis of the scaling properties of suitably regularized models. For random paths it turns out to be possible to analyze in detail the scaling behavior of rigidity terms in the action. We show that such terms are irrelevant perturbations (essentially as a consequence of the central limit theorem), and have only an effect on the scaling limit when the associated bare coupling constant is fine tuned to infinity. In this case, there are non-vanishing correlations between tangents to the paths, as opposed to ordinary Brownian paths. It is nevertheless possible to obtain an explicit expression for the corresponding propagator.

The situation for fluctuating surfaces is more complicated. A general framework, analogous to that for paths, can however be set up in a straightforward way, and a number of general results can be obtained. Thus, there is substantial evidence that for small values of the curvature coupling the only possible scaling limit equals that of a simple random walk, as we explain in Section 3. This result can be viewed as some sort of generalization

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of the central limit theorem to random surfaces, although as a surface theory the limit is degenerate. This phenomenon, in fact, makes the investigation of rigidity enhancing mechanisms for surfaces an important, albeit difficult issue. In particular, the question poses itself whether there exists a critical curvature coupling at which a non-degenerate scaling limit can be obtained. We also comment on this problem, but the results obtained so far are only fragmentary.

It is important to note that the models we consider are without any self-avoidance constraints, i.e. the paths and surfaces are allowed to self-intersect and overlap arbitrarily. As such, the models are not directly interpretable as representing fluctuating polymers or membranes, except in dimensions high enough to render the self-avoidance constraint irrelevant. Rather, we shall consider the models from a field- or string-theoretic point of view and regard the discretizations as regularizations of appropriate functional integrals to be explained below.

2 Random Paths with Rigidity

For definiteness we consider models of piecewise linear random paths, also called random flight models. The paths will be parameterized on the interval $[0,1]$ and, if the path $\omega : [0,1] \to R^d$ has N linear steps, the i'th step is parameterized linearly on the interval $[(i-1)/N, i/N]$ such that the path and its parameterization are uniquely fixed by the points $x_i = \omega(i/N),\ i = 0,\ldots,N$. Letting θ_i denote the angle between the i'th and the $(i+1)$'th step, i.e.

$$\theta_i = \arccos \frac{r_i \cdot r_{i+1}}{|r_i||r_{i+1}|} \ ,$$

where

$$r_i = x_{i+1} - x_i \ ,$$

we define a propagator (or two-point function) by

$$G_{\mu,\lambda}(x,y) = \sum_{N=1}^{\infty} \int \prod_{i=1}^{N-1} dx_i \exp\left\{ \sum_{i=1}^{N} \phi(|x_i - x_{i-1}|) + \lambda \sum_{i=1}^{N-1} \psi(\theta_i) + \mu N \right\} \ , \tag{1}$$

where we have set $\omega(0) = x$ and $\omega(1) = y$. Here ϕ and ψ are suitable continuous and non-negative functions to be specified more closely below,

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and μ, λ are coupling constants.

The right hand side of Eq. (1) should be viewed as an integral over all piecewise linear paths, where each path contributes with a weight $\exp[-S(\omega)]$, and the action $S(\omega)$ is given by

$$S(\omega) = \sum_{i=1}^{N} \phi(|x_i - x_{i-1}|) + \lambda \sum_{i=1}^{N-1} \psi(\theta_i) + \mu N \; . \tag{2}$$

This action is actually a gauge fixed and discretized form of the continuum action

$$S(\omega) = \int_a^b dte(t) \left\{ \phi_0 \left[\frac{|\dot{\omega}(t)|}{e(t)} \right] + \lambda_0 \psi_0 \left[\frac{k(t)}{e(t)} \right] + \mu_0 \right\} \; , \tag{3}$$

where $\omega : [a,b] \to R^d$ is now a (piecewise) smooth path and $e : [a,b] \to R_+$ is an intrinsic metric on the one-dimensional manifold ω. Moreover, k denotes the *extrinsic curvature* of ω defined as

$$k(t) = \frac{\{|\dot{\omega}|^2 |\ddot{\omega}|^2 - (\dot{\omega} \cdot \ddot{\omega})^2\}^{\frac{1}{2}}}{|\dot{\omega}|^2} \; .$$

Indeed, S is invariant under the reparametrizations

$$t' = \varphi(t), \quad \omega'(t') = \omega(t), \quad e'(t') = \frac{e(t)}{\dot{\varphi}(t)} \; ,$$

where φ is an increasing diffeomorphism between intervals. A unique parameterization is determined by fixing the parameter interval to be $[0,1]$ and the metric $e(t)$ to be constant and equal to itts total volume T. With this parameterization and ω piecewise linear the two actions (2) and (3) coincide [1], if the identification

$$T = a^2 N$$

is made, where a is a parameter with dimension of a length, to be viewed as a short distance cutoff, and ϕ and ψ are dimensionless analogs of ϕ_0 and ψ_0.

Specifying ϕ_0 and ψ_0 suitably, one obtains the most familiar form [2] of the action

$$S(\omega) = \int_a^b dt \left(\frac{1}{2} \frac{|\dot{\omega}(t)|^2}{e(t)} + \lambda_0 k(t) + \mu_0 e(t) \right) \; . \tag{4}$$

As will be seen, however, the scaling limit is essentially independent of the exact form of the functions ϕ and ψ. We shall demand that the function ψ

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vanishes at 0, but is otherwise positive in the interval $]0,\pi]$, such that non-straight paths get increasingly suppressed with increasing λ. For $\lambda = 0$, it is well known that generic paths in the continuum limit are far from being smooth, the set of such paths having Wiener measure 0. We shall see that this still holds for any fixed value of λ, whereas for $\lambda \to \infty$ we can obtain different continuum limits.

First, let us discuss the domain of convergence of Eq. (1). Because of translation invariance of $G_{\mu,\lambda}$, it is convenient to work with its Fourier transform

$$\hat{G}_{\mu,\lambda}(p) = \int dx G_{\mu,\lambda}(0,x) e^{-ip\cdot x} \tag{5}$$

and to introduce the two functions

$$F(s) = \int_0^\infty dr r^{d-1} e^{-\phi(r)} e^{-isr} \tag{6}$$

and

$$K_\lambda(\hat{r},\hat{r}') = e^{-\lambda\psi(\theta(\hat{r},\hat{r}'))} \ ,$$

where $s \in R$, $\hat{r},\hat{r}' \in S^{d-1}$ and $\theta(\hat{r},\hat{r}')$ denotes the angle between $\hat{r}$ and $\hat{r}'$. We consider $K_\lambda(\hat{r},\hat{r}')$ as the kernel of an integral operator K_λ on $L^2(S^{d-1})$, and for fixed $p \in R^d$ we let F_p denote multiplication by the function $F(p\cdot\hat{r})$ acting on $L^2(S^{d-1})$. One can then rewrite (1) as

$$\hat{G}_{\mu,\lambda}(p) = \langle 1|(1 - e^{-\mu} F_p K_\lambda)^{-1} e^{-\mu} F_p|1\rangle \ , \tag{7}$$

where $\langle\cdot|\cdot\rangle$ is the inner product in $L^2(S^{d-1})$ with respect to the uniform measure $d\Omega$ and where $|1\rangle$ is the constant function 1 on S^{d-1}. Henceforth, we assume that ϕ increases sufficiently fast such that F is finite and, moreover, twice continuously differentiable.

Using that $|1\rangle$ is an eigenfunction of K_λ with eigenvalue

$$\|K_\lambda\| = \int d\Omega(\hat{r}') e^{-\lambda\psi(\theta(\hat{r},\hat{r}'))},$$

one finds that the *susceptibility*

$$\chi(\mu,\lambda) = \hat{G}(\mu,\lambda)(0)$$

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is given by

$$\chi(\mu,\lambda) = \frac{\omega_d e^{-\mu} F_0}{1 - e^{-\mu}\|K_\lambda\| F_0} \ , \tag{8}$$

where now $F_0 = F(0)$ and ω_d is the volume of S^{d-1}.

It follows that $\chi(\mu,\lambda)$ and hence also $\hat{G}_{\mu,\lambda}(p)$ is finite for all $p \in R^d$ exactly, if $e^\mu > \|K_\lambda\| F_0$, i.e. the critical line in the (μ,λ)-plane is given by

$$e^{\mu_0} = \|K_\lambda\| F_0 \ , \tag{9}$$

which defines the critical coupling $\mu_0(\lambda)$ as a function of the extrinsic curvature coupling λ.

Now, let us first fix λ and consider $\hat{G}_{\mu,\lambda}(ap)$ for μ close to but larger than $\mu_0(\lambda)$, where we have replaced p by ap so that p, from now on, is a dimensionful (physical) momentum variable. Expanding

$$F(ap\cdot\hat{r}) = F_0(1 + ic_1(p\cdot\hat{r})a - c_2(p\cdot\hat{r})^2 a^2) + O(a^3) \tag{10}$$

and

$$K_\lambda = \|K_\lambda\|(P + R_\lambda) \ ,$$

where c_1, c_2 are positive constants and P denotes the projection onto the constant modes on S^{d-1}, one shows that $\|R_\lambda\| < 1$, and hence, by Eq. (7), that the singularity of $\hat{G}_{\mu,\lambda}(ap)$ as a function of μ close to $\mu_0(\lambda)$ is solely due to the largest eigenvalue of the leading term in the operator $e^{-\mu}F_p K_\lambda$ approaching 1. Moreover, since $\langle 1|(p\cdot\hat{r})1\rangle = 0$ one finds that only the second-order term in Eq. (10) contributes in Eq. (7) for small a. To obtain a non-trivial limit, this forces us to set (to leading order)

$$\mu(a) = \mu_0(\lambda) + m_1^2 a^2 \tag{11}$$

for some positive constant m_1^2. Then the denominator in Eq. (7) is of order a^2 and it follows [1,3] that

$$\lim_{a\to 0} a^2 \hat{G}_{\mu,\lambda}(ap) = \frac{c}{m^2 + p^2} \ , \tag{12}$$

where c and m^2 are positive constants.

This proves our first claim that the scaling limit of $G_{\mu,\lambda}(x,y)$ equals the free scalar propagator, regardless of the value of λ.

Next, we consider the possibility of letting $\lambda \to \infty$ and thus forcing $\mu \to \infty$ according to Eq. (9). In this limit one finds that all eigenvalues of $\|K_\lambda\|^{-1} K_\lambda$

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tend to 1, and hence they all contribute in Eq. (7). More precisely, one finds [1]

$$K_\lambda = \|K_\lambda\|[1 + C(\lambda)L + o(C(\lambda))] ,$$

where L is the Laplace-Beltrami operator on $L^2(S^{d-1})$ and $C(\lambda)$ is a positive function tending to 0 as $\lambda \to \infty$. Since all eigenvalues of K_λ contribute on the right hand side of Eq. (7), the contribution from F_p is from the first-order term in Eq. (10). Hence, we set

$$\mu(a) = \mu_0(a) + c_1\mu_R a \tag{13}$$

and choose $\lambda(a)$ such that

$$C(\lambda(a)) = c_1\lambda_R a ,$$

where μ_R , $\lambda_R > 0$ are renormalized coupling constants. With this choice one gets

$$\lim_{a\to 0} a\hat{G}_{\mu,\lambda}(ap) = \langle 1|(\mu_R + \lambda_R L - ip\cdot\hat{r})^{-1}|1\rangle .$$

In fact, more generally, if the directions of the first and last steps are held fixed at $\hat{r}'$ and $\hat{r}''$, respectively, one gets for the scaling limit of the corresponding two-point function $\hat{G}_{\mu,\lambda}(p;\hat{r}',\hat{r}'')$ the result

$$\lim_{a\to 0} a\hat{G}_{\mu,\lambda}(ap;\hat{r}',\hat{r}'') = \langle \hat{r}'|(\mu_R + \lambda_R L - ip\cdot\hat{r})^{-1}|\hat{r}''\rangle . \tag{14}$$

This fact explicitly entails the non-vanishing correlation between tangents to the paths, and hence also implicitly shows that the paths have acquired a rigidity represented by the coupling constant λ_R. It is an interesting open problem to identify the measure on the space of paths associated with this limit and to analyze in more detail the smoothness properties of generic paths. It can be argued [1,3] that their Hausdorff dimension is 1, whereas for standard Wiener paths it is known to be 2. In fact, this difference is intimately connected to the different scalings used in Eqs. (11) and (13) via scaling relations.

It is worth noting that the operator method sketched above can be applied to other types of models of fluctuating paths as well, such as the Ornstein-Uhlenbeck process [3].

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3 Random Surfaces with Rigidity

In order to extend the framework of the preceding section to random surfaces we first describe the analogue of the action functional (3).

Let $X : D \to R^d$ be a parameterized surface in R^d defined on some fixed oriented parameter domain D. By $n_1, \ldots, n_{d-2}$ we denote an oriented orthonormal basis in the normal bundle to X (defined locally), and we let

$$D_{a;ij} = \delta_{ij}\partial_a + n_i \cdot \partial_a n_j$$

be the covariant derivative in the normal bundle. Then the *extrinsic curvature* H of X is defined by

$$H^2 = \frac{1}{4} h^{ab} \sum_i D_a n_i \cdot D_b n_i \ ,$$

where h_{ab} is the first fundamental form of X.

A natural generalization of Eq. (3) is therefore

$$S(X, g_{ab}) = \int_D d^2\xi \sqrt{g} \Bigg\{ \phi_0 \left(g^{ab} \partial_a X \cdot \partial_b X \right) + \lambda_0 \psi_0 \left(g^{ab} \sum_i D_a n_i \cdot D_b n_i \right) + \mu_0 \Bigg\} \ , \tag{15}$$

where g_{ab} is an intrinsic metric defined on D, and g is its determinant. Obviously, S is invariant under reparametrizations

$$\xi' = \varphi(\xi), \quad X'(\xi') = X(\xi), \quad g'_{ab}(\xi')\partial_c \varphi^a(\xi) \partial_d \varphi^b(\xi) = g_{cd}(\xi) \ ,$$

where φ is an orientation preserving diffeomorphism between parameter domains.

Specifying ϕ_0 and ψ_0 suitably, one obtains the most familiar form [4–6] of the action

$$S(X, g_{ab}) = \int_D d^2\xi \sqrt{g} \left(\frac{1}{2} g^{ab} \partial_a X \cdot \partial_b X + \lambda_0 g^{ab} \sum_i D_a n_i \cdot D_b n_i + \mu_0 \right) \ . \tag{16}$$

To obtain a discretized form of S, we use the general idea [1] to consider piecewise linear surfaces defined as maps from the vertices of an (oriented) triangulation of D into R^d, and to assign to each such triangulation T the

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metric that assigns equal length a to all links and is Euclidean on each triangle. This leads us to set [1,7]

$$S_T(X) = \sum_{(ij)} \left[\phi(|x_i - x_j|) + \lambda\psi(\theta_{(ij)})\right] + \mu|T| \;, \tag{17}$$

where the sum is over nearest-neighbouring pairs of vertices (i.e. links) in T, x_i is the image of the vertex i in R^d, $\theta_{(ij)}$ is the angle between the normals to the triangles sharing (ij) in the three-dimensional space spanned by the two triangles, and $|T|$ is the number of triangles in T.

The discretized model is then defined in terms of its *loop correlation functions*

$$G_{\mu,\lambda}(\gamma_1,\ldots,\gamma_n) = \sum_{T\in\mathcal{T}(m_1,\ldots,m_n)} \int \prod_{i\in T\setminus\partial T} dx_i \exp[-S_T(X)] \;, \tag{18}$$

where $\gamma_1,\ldots,\gamma_n$ are polygonal loops in R^d, having $m_1,\ldots,m_n$ vertices, respectively, and $\mathcal{T}(m_1,\ldots,m_n)$ denotes the set of abstract triangulations with n boundary components, having $m_1,\ldots,m_n$ vertices, respectively, that are assumed to be mapped onto $\gamma_1,\ldots,\gamma_n$ in the obvious way, and, of course, integration is over interior vertices only. Interpreting the vertices in T as locations of molecules and the first term in the action as representing pair interactions between nearest-neighbouring molecules, the randomness of T means that a given molecule does not have fixed neighbours. Therefore the model is usually referred to as a *fluid membrane* model as opposed to *crystalline membranes* where T is fixed. Both types of models have a broad variety of potential application ranging from biophysics to high-energy physics [8].

It is important to note first that the triangulations T have to be topologically constrained, since otherwise the sum in (18) is manifestly divergent [1]. Thus we assume in the following that T is homeomorphic to a sphere with n holes. Under similar assumptions on the functions ϕ and ψ as in the previous section one shows [1] that the loop correlation functions $G_{\mu,\lambda}$ are well defined and finite in a convex domain $\mathcal{B}$ in the (μ,λ)-plane, independent of the number of loops and their shape.

The boundary of $\mathcal{B}$ defines the critical line $\mu = \mu_0(\lambda)$ above which the loop correlation functions are finite. To discuss the critical behavior, it is useful to introduce the following three quantities: the *susceptibility*

$$\chi(\mu,\lambda) = \int_{R^d} dx G_{\mu,\lambda}(\gamma_0,\gamma_x) \;,$$

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where γ_x denotes the degenerate loop consisting of the single point x. The *mass gap*

$$m(\mu,\lambda) = -\lim_{|x|\to\infty} \frac{1}{|x|} \log G_{\mu,\lambda}(\gamma_0,\gamma_x) \ ,$$

and the *string tension*

$$\tau(\mu,\lambda) = -\lim_{R\to\infty} \frac{1}{R} \log G_{\mu,\lambda}(\gamma_{R\times R}) \ ,$$

where $\gamma_{R\times R}$ is a square loop with sides of length R and with a number of equidistant vertices proportional to R.

Of course, the domain $\mathcal{B}$ and the functions $G_{\mu,\gamma}$ depend on the functions ϕ and ψ, but it is expected that the critical behavior of the loop correlation functions is essentially independent of the details of those functions. As it turns out, however, the concrete analysis of the model is for many purposes most conveniently carried out for the discretized version of (16),

$$S_T(X) = \sum_{(ij)} \left(\frac{1}{2}|x_i - x_j|^2 + \lambda(1-\cos\theta_{(ij)}) \right) + \mu|T| \ . \tag{19}$$

Thus, in this case, the extrinsic curvature contribution from the link shared by the triangles Δ, Δ' is simply

$$\frac{1}{2}\lambda(n_\Delta - n_{\Delta'})^2 = \lambda(1 - n_\Delta \cdot n_{\Delta'}) \ ,$$

where n_Δ is the unit normal vector to Δ in the space spanned by Δ and Δ', and similarly for $n_{\Delta'}$. The results alluded to in the following mostly refer to this particular model.

For $\lambda = 0$ there is ample evidence [1] that the scaling limit is a free scalar field. To explain this statement in more detail, we first recall that a necessary condition for the existence of a scaling limit is that $m(\mu,\lambda) \to 0$ as $\mu \to \mu_0(\lambda)$, and in this case the physical mass m_0 and the scaling of μ (for fixed λ) is defined by

$$m(\mu,\lambda) = m_0 a \ . \tag{20}$$

Moreover, for a scaling limit of genuine continuum surfaces one would expect that $\tau(\mu,\lambda) \to 0$ as $\mu \to \mu_0(\lambda)$ such that the physical string tension τ_0, defined by

$$\tau(\mu(a),\lambda) = \tau_0 a^2 \ , \tag{21}$$

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is finite. It turns out [1], however, for $\lambda = 0$, that $\tau(\mu, 0)$ approaches a positive constant as $\mu \to \mu_0(0)$, implying an infinite continuum string tension. As a consequence, the dominant surfaces are collapsed ones with spiky outgrowths, effectively branched polymer-like structures. Furthermore, the scaling limit of the two-point function $G_{\mu,0}(\gamma_0, \gamma_x)$, defined as in Eq. (12), exists and is again proportional to the free scalar propagator. A thorough discussion of these results and in particular the role of branched polymers can be found in Ref. [1].

The question arises whether this so-called branched polymer phase or crumpled phase of the model persists for all values of λ, as was shown for the random walk model of the previous section. This is still an open problem, at least from a rigorous point of view within the setup sketched above. However, the framework allows for efficient numerical simulations [1]. Most of these seem to indicate the existence of a critical value $\lambda_c > 0$ at which both the string tension and the mass scale tend to zero, and the possibility of the existence of a trajectory approaching $(\mu_0(\lambda_c), \lambda_c)$ along which the ratio m^2/τ stays constant, as demanded by (20) and (21), is not excluded.

On the other hand, perturbative expansions in $1/\lambda$ have been carried out [9–13]. To lowest order it is found that the β-function is negative for small $1/\lambda$, and it is conceivable that there is no further zero, but this has so far not been established.

Of course, it is of considerable interest to decide firmly which of the two scenarios occurs, and, in case of the latter, whether it is possible by fine tuning λ to ∞ to obtain a non-trivial scaling limit corresponding to (14). This is a problem left for future research.

4 Conclusions

We have given an outline of results on the scaling properties of a class of discretized models of fluctuating paths and surfaces with rigidity. The discretizations considered here are, of course, by no means unique, and a variety of others have been considered. In particular, hypercubic models have played an important role and yield results complementary to the ones discussed in the text. We refer to the literature [1] for details.

Clearly, this article is by no means intended to be an exhaustive overview of the theory of random surfaces, which is a field in rapid development. In particular, the whole subject of crystalline membranes, which have been investigated by both analytical and numerical methods in recent years, has

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been left out, and so have self-avoiding membranes. We refer the reader to a recent review [8] for an account of its fascinating aspects.

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