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# CONFINEMENT IN THE ENSEMBLES OF MONOPOLES

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String representations of the Wilson loop in three-dimensional gases of $SU(2)$ and $SU(3)$ Abelian-projected monopoles are discussed. It is demonstrated that the summation over world sheets bounded by the contour of the Wilson loop is realized by summing over branches of a certain effective multi-valued potential of monopole densities. Finally, by virtue of the so-constructed representation of the Wilson loop, this quantity is evaluated in the $SU(2)$-inspired case within the approximation of a dilute monopole gas, which makes confinement in the model under study manifest.

## 1 Introduction

On the way of constructing the string representation of QCD by means of the method of Abelian projections [1], the main results have been obtained under the assumption of the monopole condensation. Such condensation can be described by demanding that fluctuating monopole trajectories, forming the grand canonical ensemble, possess several natural properties. These properties, which can be elegantly formulated in the path-integral language [2,3], are the presence of the kinetic and mass terms of a trajectory, as well as the short-range interaction of the trajectories.

Another way to model the monopole condensation is to consider the grand canonical ensemble of monopoles as a Coulomb gas [2,4]. The respective $SU(2)$-inspired theory then turns out to be compact QED, i.e. electrodynamics with monopoles. The novel type of gauge invariance appearing in this

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theory (the so-called monopole gauge invariance) and its condensed matter analogues have been discovered by Professor Hagen Kleinert in Ref. [5]. The consistent local quantum field theory of electrically charged particles and monopoles has for the first time been constructed in Refs. [5,6] and discussed in details in Ref. [7]. A very important result of these investigations, which has then been used many times in the literature on the dual models of confinement, is that the Wilson loop in this theory remains invariant under the duality transformation.

In what follows, we shall just consider the grand canonical ensembles of $SU(2)$ and $SU(3)$ Abelian-projected monopoles in 2+1 dimensions and string representations of the Wilson loop in the respective disorder field theories. In Section 2, we shall consider the simplest $SU(2)$-inspired case (i.e. compact QED) and then, in Section 3, we will extend this analysis to the $SU(3)$-inspired theory. The main results of our study will be summarized in the conclusions.

## 2 String Representation of the Wilson Loop in Compact QED

The action of the Coulomb gas of monopoles in 3D compact QED has the form

$$S_{\mathrm{mon}} = g_m^2 \sum_{a<b} q_a q_b \left(\Delta^{-1}\right)(\mathbf{z}_a, \mathbf{z}_b) + S_0 \sum_a q_a^2. \tag{1}$$

Here, $\Delta$ is the 3D Laplace operator, and $S_0$ is the action of a single monopole, $S_0 = \mathrm{const.}/e^2$. We have also adopted the standard Dirac notations, where $eg_m = 2\pi n$, restricting ourselves to the monopoles of the minimal charge, i.e. setting $n = 1$. Then, the partition function of the grand canonical ensemble of monopoles corresponding to the action (1) reads

$$\mathcal{Z}_{\mathrm{mon}} = 1 + \sum_{N=1}^{\infty} \sum_{q_a=\pm 1} \frac{\zeta^N}{N!} \prod_{i=1}^{N} \int d^3 z_i \times \exp\left[-\frac{g_m^2}{8\pi} \int d^3x d^3y \rho_{\mathrm{gas}}(\mathbf{x}) \frac{1}{|\mathbf{x}-\mathbf{y}|} \rho_{\mathrm{gas}}(\mathbf{y})\right], \tag{2}$$

where

$$\rho_{\mathrm{gas}}(\mathbf{x}) = \sum_{a=1}^{N} q_a \delta\left(\mathbf{x} - \mathbf{z}_a\right)$$

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is the density of the monopole gas. Here, a single monopole weight $\zeta \propto \exp(-S_0)$ has the dimension of $(\text{mass})^3$. It is usually referred to as the fugacity. Notice also that we have restricted ourselves to the values $q_a = \pm 1$, since monopoles with $|q| > 1$ turn out to be unstable and tend to dissociate into those with $|q| = 1$. That is because the energy of a single monopole is a quadratic function of its flux. Thus it is energetically more favourable for the vacuum to support a configuration of two monopoles of the unit magnetic charge than one monopole of the double charge.

Next, Coulomb interaction can be made local, albeit a nonlinear one, by introducing an auxiliary scalar field:

$$\mathcal{Z}_{\text{mon}} = \int D\chi \exp\left\{-\int d^3x \left[\frac{1}{2}(\nabla\chi)^2 - 2\zeta\cos(g_m\chi)\right]\right\}. \tag{3}$$

The magnetic mass $m = g_m\sqrt{2\zeta}$ of the dual boson $\chi$, following from the quadratic term in the expansion of the cosine on the r.h.s. of Eq. (3), is due to the Debye screening of this boson in the monopole gas.

Let us now cast the partition function (2) into the form of an integral over monopole densities. This can be done by introducing into Eq. (2) a unity of the form

$$1 = \int D\rho\, \delta\left(\rho(\mathbf{x}) - \rho_{\text{gas}}(\mathbf{x})\right) = \int D\rho D\lambda \exp\left\{i\left[\sum_{a=1}^{N} q_a\lambda(\mathbf{z}_a) - \int d^3x\lambda\rho\right]\right\}. \tag{4}$$

Then, performing the summation over the monopole ensemble in the same way as it has been done in a derivation of the representation (3), we get

$$\mathcal{Z}_{\text{mon}} = \int D\rho D\lambda \exp\left\{-\frac{g_m^2}{8\pi}\int d^3x d^3y \rho(\mathbf{x})\frac{1}{|\mathbf{x}-\mathbf{y}|}\rho(\mathbf{y}) + \right.$$
$$\left. + \int d^3x\left(2\zeta\cos\lambda - i\lambda\rho\right)\right\}. \tag{5}$$

Finally, integrating over the Lagrange multiplier $\lambda$ by resolving the corresponding saddle-point equation,

$$\sin\lambda = -\frac{i\rho}{2\zeta}, \tag{6}$$

we arrive at the following expression for the partition function

$$\mathcal{Z}_{\text{mon}} = \int D\rho \exp\left\{-\left[\frac{g_m^2}{8\pi}\int d^3x d^3y \rho(\mathbf{x})\frac{1}{|\mathbf{x}-\mathbf{y}|}\rho(\mathbf{y}) + V[\rho]\right]\right\}. \tag{7}$$

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Here,

$$V[\rho] = \int d^3x \left[\rho \operatorname{arcsinh}\left(\frac{\rho}{2\zeta}\right) - 2\zeta\sqrt{1+\left(\frac{\rho}{2\zeta}\right)^2}\right] \tag{8}$$

is the effective multi-valued potential of monopole densities. Due to Eq. (4), the obtained representation (7) is natural to be called as a representation for the partition function in terms of the monopole densities.

We are now in the position to discuss the string representation of the Wilson loop in 3D compact QED. Such a loop is nothing else, but an averaged phase factor of an electrically charged test particle propagating along a closed trajectory $C$. Since the Wilson loop depends only on this trajectory, the desired string representation should be understood as some mechanism providing the independence of the loop from a certain string world sheet $\Sigma$ bounded by $C$. By virtue of the Stokes theorem, the Wilson loop can be rewritten in the following form

$$\begin{aligned}\langle W(C)\rangle &= \left\langle \exp\left(\frac{i}{2}\int\limits_{\Sigma} d\sigma_{\mu\nu}\mathcal{F}_{\mu\nu}\right)\right\rangle_{A_\mu,\rho} \\ &= \langle W(C)\rangle_{A_\mu}\left\langle \exp\left(\frac{i}{2}\int d^3x\rho(\mathbf{x})\eta(\mathbf{x})\right)\right\rangle_{\rho}. \end{aligned} \tag{9}$$

Here, $\langle W(C)\rangle_{A_\mu}$ stands for the standard free-photon contribution, whereas the average over monopole densities is defined by the partition function (7). We have also defined by $\mathcal{F}_{\mu\nu} \equiv \mathcal{F}_{\mu\nu}[\rho]$ the full electromagnetic field strength tensor $F_{\mu\nu} + F^M_{\mu\nu}$, which includes the monopole part $F^M_{\mu\nu}[\rho]$ obeying the modified Bianchi identities

$$\frac{1}{2}\varepsilon_{\mu\nu\lambda}\partial_\mu F^M_{\nu\lambda} = 2\pi\rho.$$

Also, in Eq. (9),

$$\eta(\mathbf{x}) = \partial^x_\mu \int\limits_{\Sigma} d\sigma_\mu(\mathbf{y})\frac{1}{|\mathbf{x}-\mathbf{y}|}$$

stands for the solid angle under which the surface $\Sigma$ shows up to an observer located at point $\mathbf{x}$.

Equation (9) seems to contain some discrepancy, since its l.h.s. depends only on the contour $C$, whereas the r.h.s. depends on an arbitrary surface

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$\Sigma$ bounded by $C$. However, this actually occurs to be not a discrepancy, but a key point in the construction of the desired string representation. The resolution of the apparent paradox lies in the observation that the surface independence is realized by summing over all complex-valued branches of the monopole potential (8) at every space point $\mathbf{x}$.

It is worth noting that the so-obtained string representation (9) has been derived for a first time in another, more indirect, way in Ref. [8]. It is therefore instructive to establish a correspondence between our approach and the one of that paper.

The main idea of Ref. [8] was to calculate the Wilson loop starting with the direct definition of this average in the sense of the partition function (2) of the monopole gas. The respective expression has the form

$$\begin{aligned}\langle W(C)\rangle_{\rm mon} &= 1+\sum_{N=1}^{\infty}\sum_{q_a=\pm 1}\frac{\zeta^N}{N!}\prod_{i=1}^{N}\int d^3z_i \exp\left[-\frac{g_m^2}{8\pi}\int d^3x d^3y\right.\\ &\left.\times\rho_{\rm gas}(\mathbf{x})\frac{1}{|\mathbf{x}-\mathbf{y}|}\rho_{\rm gas}(\mathbf{y})+\frac{i}{2}\int d^3x\rho_{\rm gas}(\mathbf{x})\eta(\mathbf{x})\right]\\ &= \int D\varphi \exp\left\{-\int d^3x\left[\frac{e^2}{8\pi^2}\left(\partial_\mu\varphi-\frac{1}{2}\partial_\mu\eta\right)^2-2\zeta\cos\varphi\right]\right\}, \qquad (10)\end{aligned}$$

where $\varphi \equiv g_m\chi + \eta/2$.

Next, one can prove the following equality

$$\begin{aligned}&\exp\left[-\frac{e^2}{8\pi}\oint_C dx_\mu \oint_C dy_\mu \frac{1}{|\mathbf{x}-\mathbf{y}|}-\frac{e^2}{8\pi^2}\int d^3x\left(\partial_\mu\varphi-\frac{1}{2}\partial_\mu\eta\right)^2\right]\\ &= \int Dh_{\mu\nu}\exp\left[-\int d^3x\left(i\varphi\varepsilon_{\mu\nu\lambda}\partial_\mu h_{\nu\lambda}+g_m^2h_{\mu\nu}^2-2\pi i h_{\mu\nu}\Sigma_{\mu\nu}\right)\right], \qquad (11)\end{aligned}$$

which makes it possible to represent the contribution of the kinetic term on the r.h.s. of Eq. (10) and the free-photon contribution to the Wilson loop as an integral over the Kalb-Ramond field $h_{\mu\nu}$. The only nontrivial point necessary to prove the equality (11) is an expression for the derivative of the solid angle [2],

$$\partial_\lambda\eta(\mathbf{x}) = \varepsilon_{\lambda\mu\nu}\partial_\mu^x\oint_C dy_\nu\frac{1}{|\mathbf{x}-\mathbf{y}|}-4\pi\int_\Sigma d\sigma_\lambda(\mathbf{y})\delta(\mathbf{x}-\mathbf{y}).$$

Making use of this result and carrying out the Gaussian integral over the field

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$h_{\mu\nu}$, one can demonstrate that both sides of Eq. (11) are equal to

$$\exp\left\{-\frac{e^2}{2}\left[\frac{1}{4\pi^2}\int d^3x\,(\partial_\mu\varphi)^2+\frac{1}{\pi}\int\limits_\Sigma d\sigma_\mu\partial_\mu\varphi+\right.\right.$$
$$\left.\left.+\int\limits_\Sigma d\sigma_\mu(\mathbf{x})\int\limits_\Sigma d\sigma_\mu(\mathbf{y})\delta(\mathbf{x}-\mathbf{y})\right]\right\},$$

thus proving the validity of this equation.

Substituting now Eq. (11) into Eq. (10), it is straightforward to carry out the integral over the field $\varphi$. Since this field has no more kinetic term, such an integration can be performed in the saddle-point approximation. The respective saddle-point equation has the same form as Eq. (6) with the replacement $\rho\to\varepsilon_{\mu\nu\lambda}\partial_\mu h_{\nu\lambda}$. As a result, we obtain the following expression for the full Wilson loop

$$\langle W(C)\rangle=\langle W(C)\rangle_{A_\mu}\langle W(C)\rangle_{\rm mon}=\int Dh_{\mu\nu}$$
$$\times\exp\left\{-\int d^3x\left(g_m^2h_{\mu\nu}^2+V\left[\varepsilon_{\mu\nu\lambda}\partial_\mu h_{\nu\lambda}\right]\right)+2\pi i\int\limits_\Sigma d\sigma_{\mu\nu}h_{\mu\nu}\right\},\quad(12)$$

where the world-sheet independence of the r.h.s. is again provided by summing over branches of the multi-valued action, which is now the action of the Kalb-Ramond field $h_{\mu\nu}$.

Comparing Eqs. (9) and (12), we see that the Kalb-Ramond field is indeed related to the monopole density via the equation $\varepsilon_{\mu\nu\lambda}\partial_\mu h_{\nu\lambda}=\rho$. Thus, we see that the same Legendre transformation which made out of $\rho_{\rm gas}(\mathbf{x})$ the dynamical field $\rho(\mathbf{x})$ transforms the field $\mathcal{F}_{\mu\nu}\left[\rho_{\rm gas}\right]/(4\pi)$ to the dynamical Kalb-Ramond field $h_{\mu\nu}$. In the formal language, such a decomposition of the Kalb-Ramond field is just the essence of the Hodge decomposition theorem.

Let us now consider the case of a very dilute monopole gas $|\rho|\ll\zeta$, and restrict ourselves to the real branch of the effective potential. This yields the following expression for the Wilson loop

$$\langle W(C)\rangle_{\rm dil.\,gas}=\int Dh_{\mu\nu}\exp\left\{-\int d^3x\left[\frac{1}{6\zeta}H_{\mu\nu\lambda}^2+g_m^2h_{\mu\nu}^2-2\pi ih_{\mu\nu}\Sigma_{\mu\nu}\right]\right\},\quad(13)$$

where $H_{\mu\nu\lambda}=\partial_\mu h_{\nu\lambda}+\partial_\lambda h_{\mu\nu}+\partial_\nu h_{\lambda\mu}$ is the Kalb-Ramond field strength tensor. Notice that the mass of the Kalb-Ramond field stemming from this

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equation is equal to the Debye mass $m$ of the field $\chi$ from Eq. (3).

Clearly as we now see by restricting ourselves to the real branch of the potential we have violated the surface independence of the r.h.s. of Eq. (13). This independence can be restored by replacing $\Sigma$ by the surface of the minimal area, $\Sigma_{\rm min} \equiv \Sigma_{\rm min}[C]$. After that, the quantity

$$S_{\rm str} = -\ln \langle W(C) \rangle_{\rm dil.\, gas} \Big|_{\Sigma \to \Sigma_{\rm min}} \tag{14}$$

can be considered as a string effective action of 3D compact QED in the dilute monopole gas approximation.

Integrating over the Kalb-Ramond field, we get (apart from the boundary term)

$$S_{\rm str} = \frac{\pi\zeta}{2} \int\limits_{\Sigma_{\rm min}} d\sigma_{\mu\nu}(\mathbf{x}) \int\limits_{\Sigma_{\rm min}} d\sigma_{\mu\nu}(\mathbf{y}) \frac{{\rm e}^{-m|\mathbf{x}-\mathbf{y}|}}{|\mathbf{x}-\mathbf{y}|}.$$

This non-local action can be further expanded in powers of the derivatives with respect to the world-sheet coordinates $\xi = (\xi^1, \xi^2)$, which is equivalent to the $1/m$-expansion. Then, as the first two non-trivial terms, we get the Nambu-Goto and the Polyakov-Kleinert terms [9,10]:

$$S_{\rm str} \simeq \sigma \int d^2\xi \sqrt{g} + \frac{1}{\alpha} \int d^2\xi \sqrt{g} g^{ab} (\partial_a t_{\mu\nu})(\partial_b t_{\mu\nu}). \tag{15}$$

Here, $g^{ab} = (\partial^a x_\mu)(\partial^b x_\mu)$ is the induced metric tensor corresponding to the world sheet $\Sigma(C)$ parameterized by the vector $x_\mu(\xi)$, $g$ is the determinant of this tensor, and $t_{\mu\nu} = \varepsilon^{ab}(\partial_a x_\mu)(\partial_b x_\nu)/\sqrt{g}$ is the extrinsic curvature tensor corresponding to the same world sheet. The Polyakov-Kleinert term describes the stiffness of the string and makes the obtained local string action much more suitable for modelling the QCD string than the pure Nambu-Goto action [11,12]. The string tension and the inverse coupling constant of the Polyakov-Kleinert term read $\sigma = \pi^2\sqrt{2\zeta}/g_m$ and $1/\alpha = -\pi^2/(8\sqrt{2\zeta} g_m^3)$, respectively. Both of them are non-analytic in the electric coupling constant, which manifests the non-perturbative nature of confinement in the model under study. Notice also that the negative sign of $\alpha$ is important for providing the stability of the string [10,12].

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## 3 String Representation of the Wilson Loop in the Gas of $SU(3)$ Abelian-Projected Monopoles

The partition function describing the grand canonical ensemble of $SU(3)$ Abelian-projected monopoles has the form [13]

$$\mathcal{Z}_{\rm mon} = 1 + \sum_{N=1}^{\infty} \frac{\zeta^N}{N!} \left( \prod_{a=1}^{N} \int d^3 z_a \sum_{\alpha_a = \pm 1, \pm 2, \pm 3} \right) \exp\left[ -\frac{g_m^2}{4\pi} \sum_{a<b} \frac{\vec{q}_{\alpha_a} \vec{q}_{\alpha_b}}{|\mathbf{z}_a - \mathbf{z}_b|} \right]. \tag{16}$$

Here, the magnetic coupling constant $g_m$ is related to the QCD coupling constant $g$ according to the equation $gg_m = 4\pi$, $\zeta \propto \exp\left(-{\rm const.}/g^2\right)$ is again the fugacity of a single monopole, and $\vec{q}_{\alpha_a}$'s are the nonzero weights of the zero triality adjoint representation of ${}^*SU(3)$. These weights are defined as $\vec{q}_1 = (1/2, \sqrt{3}/2)$, $\vec{q}_2 = (-1, 0)$, $\vec{q}_3 = (1/2, -\sqrt{3}/2)$, $\vec{q}_{-\alpha} = -\vec{q}_\alpha$. Notice that for every $\alpha = \pm 1, \pm 2, \pm 3$ and $\vec{\lambda} = (\lambda_3, \lambda_8)$ [where in the Gell-Mann basis $\lambda_3 = {\rm diag}\,(1, -1, 0)$, $\lambda_8 = {\rm diag}\,\left(1/\sqrt{3}, 1/\sqrt{3}, -2/\sqrt{3}\right)$], the following relation holds: $\vec{q}_\alpha \vec{\lambda} = \hat{n}$. Here, $\hat{n}$ is some traceless diagonal matrix with the elements $0, \pm 1$. This matrix can thus be written as $\hat{n} = w\lambda_3 w^{-1}$, where $w$ is any of the six elements of the permutation group $S_3$, which is the reason for the same amount of vectors $\vec{q}_{\pm\alpha}$'s.

Equation (16) can be represented as

$$\mathcal{Z}_{\rm mon} = \int D\vec{\chi} \exp\left\{ -\int d^3 x \left[ \frac{1}{2} \left(\nabla \vec{\chi}\right)^2 - 2\zeta \sum_{\alpha=1}^{3} \cos\left(g_m \vec{q}_\alpha \vec{\chi}\right) \right] \right\}, \tag{17}$$

and we see that the property $\vec{q}_{-\alpha} = -\vec{q}_\alpha$ yields in this case the cosine in the action rigorously. Let us now define the Wilson loop as

$$\langle W(C) \rangle \equiv \frac{1}{3} \left\langle {\rm tr}\, P \, \exp\left( \frac{i}{2} \oint\limits_C dx_\mu \vec{A}_\mu \vec{\lambda} \right) \right\rangle \tag{18}$$

with $\vec{A}_\mu \equiv (A_\mu^3, A_\mu^8)$ and consider the monopole contribution to this quantity in the theory (17). Then, it turns out that we have the following string representation [14] [compare with Eqs. (7)-(9)]:

$$\langle W(C) \rangle_{\rm mon} = \frac{1}{3\mathcal{Z}_{\rm mon}} \sum_{\alpha=1}^{3} \int D\vec{\rho} \exp\left\{ -\left[ \frac{g_m^2}{8\pi} \int d^3 x d^3 y \vec{\rho}(\mathbf{x}) \frac{1}{|\mathbf{x} - \mathbf{y}|} \vec{\rho}(\mathbf{y}) \right.\right.$$

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$$+\sum_{\beta=1}^{3} V[\rho_\beta] - i\int d^3x \vec{\rho}\vec{\eta}_\alpha \eta\Bigg]\Bigg\}. \tag{19}$$

Here, we have introduced the vectors

$$\vec{\eta}_1 = \left(-\frac{1}{2}, \frac{1}{2\sqrt{3}}\right),\ \vec{\eta}_2 = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right),\ \vec{\eta}_3 = \left(0, -\frac{1}{\sqrt{3}}\right),$$

which are just the weights of the representation **3** of $^*SU(3)$. These vectors thus determine the charges of quarks of three colours with respect to the diagonal gluons $\vec{A}_\mu$. Besides that we have denoted

$$\rho_1 = \frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{3}}\rho^1 + \rho^2\right),\ \rho_2 = -\frac{2}{3}\rho^1,\ \rho_3 = \frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{3}}\rho^1 - \rho^2\right),$$

where $\rho^{1,2}$ are the components of the monopole density, $\vec{\rho} \equiv (\rho^1, \rho^2)$.

One can see that it is again the sum over branches of the multi-valued monopole potential (8) which provides the surface independence of the r.h.s. of Eq. (19). Besides that, we see that the string representation of the Wilson loop in the $SU(3)$ monopole gas differs from that of the $SU(2)$-case. This disproves the conjecture put forward in Ref. [8], according to which the $SU(2)$-inspired action should be universal for any gauge group $SU(N)$. Finally, it is straightforward to integrate over monopole densities in the dilute gas approximation, which yields again the string effective action of the form (15).

## 4 Conclusions

In the present paper, we have derived the string representations of the Wilson loop in 2+1-dimensional gases of $SU(2)$ and $SU(3)$ Abelian-projected monopoles. Contrary to photons, monopoles do not interact with the contour of the Wilson loop, but rather with the string world sheet spanned by this contour. Therefore in both cases, the resulting string representation was understood as a certain mechanism providing the independence of the Wilson loop of the choice of such a world sheet. It has been demonstrated that such a mechanism can be based on the summation over branches of a certain multi-valued potential of monopole densities. Substituting for such densities the Kalb-Ramond field unambiguously related to them via the modified Bianchi identities, one arrives in the $SU(2)$-inspired case at the theory of confining strings, whereas in the $SU(3)$-inspired case one gets a different action.

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Although such a reformulation of the functional integral in terms of the Kalb-Ramond field allows one to account automatically also for the non-compact (photon) part of the gauge fields, it is physically less transparent than the proposed representation in terms of the monopole densities.

In the approximation of a very dilute monopole gas, the real branch of the monopole potential becomes a quadratic functional, and one can explicitly integrate monopoles out. This produces the non-local string effective action, whose gradient expansion yields the Nambu-Goto and the Polyakov-Kleinert terms as the leading ones. Those make confinement in the models under study manifest and ensure the stability of strings.

However, it remains unclear within the monopole-gas models how to derive the monopole fugacity itself from the QCD Lagrangian. Some attempts in this direction have been done in Ref. [15], but the explicit answer is unfortunately not yet found. The still pending question is: What is the proportionality coefficient between the string tension in QCD and $\Lambda^2_{\rm QCD}$? Since the answer to this question is very important for the understanding of the connection between the perturbative and non-perturbative phenomena in QCD, it requires further investigations.

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